

NAME: MODEL SOLUTIONS

Math 124 FALL 2004: Section 7 TTh 9:25-10:50 AM

Midterm 1

Date: Mar 17, 2005

Instructions: Answer all questions. Show as much work as you feel reasonable. You have 75 minutes. To allow others to fully concentrate at the end please do not leave in the last 5 minutes. You should submit your page of notes with your test paper.

Question 1. (25 points)

Suppose that you observe the following data

2.0 3.4 3.9 4.6 4.9 5.1 5.1 5.8 6.0 6.3 6.4 6.6 6.7

(a) Compute the median of this data

6pts
13 observations so $\frac{13+1}{2} = 7^{\text{th}}$ observation is median
median = 5.1

(b) Calculate the IQR.

6pts
 $LQ = \frac{3.9 + 4.6}{2} = 4.25$ $UQ = \frac{6.3 + 6.4}{2} = 6.35$

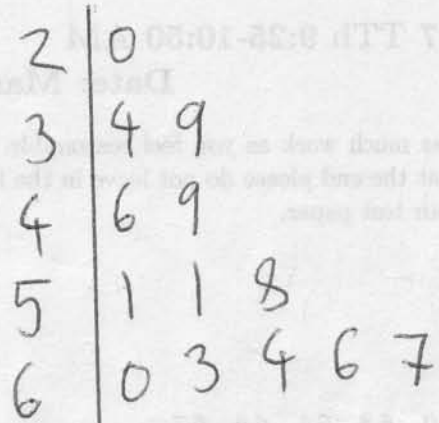
$IQR = 6.35 - 4.25 = 2.1$

(c) Identify the observations that are outliers using the 1.5IQR rule discussed in class. Make it clear how you identified these observations.

6pts
outlier if $y > UQ + 1.5 IQR = 6.35 + 3.15 = 9.5$
or $y < LQ - 1.5 IQR = 4.25 - 3.15 = 1.1$

So there are no outliers

(d) Draw and interpret the appropriate stem and leaf plot



Left (negative) skewed

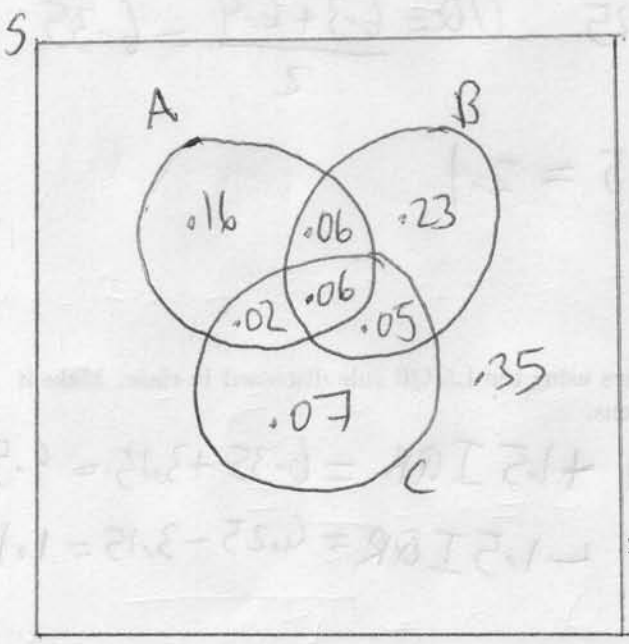
7pts

↳ this represents the decimal place

Question 2. (25 points)

Suppose you have three events A , B and C . Furthermore, you have the following probabilities:
 $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.2$, $P(A \cap C) = 0.08$, $P(B \cap C) = 0.11$, $P(A^C \cap B) = 0.28$ and
 $P(A \cap B \cap C) = 0.06$.

(a) Draw the appropriate Venn Diagram to represent all three events. Be sure to show all the probabilities.



9pts

9pts

9pts

(b) Are events A and B independent? How about B and C?

From Venn Diagram

6pts

$$P(A)P(B) = (.3)(.4) = .12 = (.06 + .06) = .12 = P(A \cap B)$$

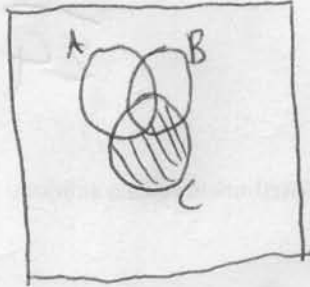
\Rightarrow so A and B are independent

$$P(B)P(C) = (.4)(.2) = .08 \neq .11 = P(B \cap C)$$

\Rightarrow so B and C are dependent (not independent)

(c) What is $P(A^c \cap C)$?

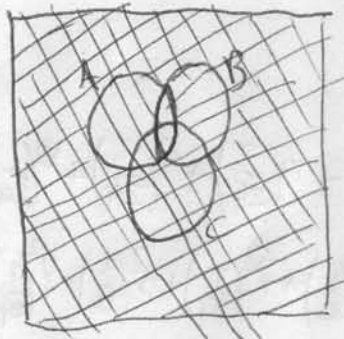
5pts



$$P(A^c \cap C) = .07 + .05 = .12$$

(d) What is $P(A^c \cup B^c)$?

5pts



$$P(A^c \cup B^c) = .16 + .23 + .02 + .05 + .07 + .35 = .88$$

$$\begin{aligned} \text{(or } P(A^c \cup B^c) &= P((A \cap B)^c) \\ &= 1 - P(A \cap B) \\ &= 1 - .12 \\ &= .88 \end{aligned}$$

Question 3. (25 points)

Imagine that you carry out a random experiment where you first roll a fair 6 sided dice, then you roll a fair 4 sided dice.

- (a) Give the sample space for this experiment. Then explain what the probability of each individual outcome will be and why.

$$S = \{ (1,1), (1,2), (1,3), (1,4), \\ (2,1), (2,2), (2,3), (2,4), \\ (3,1), (3,2), (3,3), (3,4), \\ (4,1), (4,2), (4,3), (4,4), \\ (5,1), (5,2), (5,3), (5,4), \\ (6,1), (6,2), (6,3), (6,4) \}$$

Equally likely outcomes
so probability of any outcome is $\frac{1}{24}$

8pts

- (b) Suppose that X is "the sum of the two rolls". Give the probability distribution of this random variable.

x	2	3	4	5	6	7	8	9	10
$P(X=x)$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{3}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{3}{24}$	$\frac{2}{24}$	$\frac{1}{24}$

8pts

- (c) What are the mean and standard deviations of X ?

$$\mu_X = 2\left(\frac{1}{24}\right) + 3\left(\frac{2}{24}\right) + 4\left(\frac{3}{24}\right) + 5\left(\frac{4}{24}\right) + 6\left(\frac{4}{24}\right) + 7\left(\frac{4}{24}\right) + 8\left(\frac{3}{24}\right) + 9\left(\frac{2}{24}\right) + 10\left(\frac{1}{24}\right) = \frac{144}{24} = 6$$

$$\sigma_x^2 = \sum X_i^2 P_i - \mu_X^2 = 2^2\left(\frac{1}{24}\right) + 3^2\left(\frac{2}{24}\right) + 4^2\left(\frac{3}{24}\right) + 5^2\left(\frac{4}{24}\right) + 6^2\left(\frac{4}{24}\right) + 7^2\left(\frac{4}{24}\right) + 8^2\left(\frac{3}{24}\right) + 9^2\left(\frac{2}{24}\right) + 10^2\left(\frac{1}{24}\right) - (6)^2$$

$$= \frac{964}{24} - 36 = 4.1667 \text{ (4dp)}$$

9pts

$$\sigma_x = \sqrt{4.1667} = 2.0412 \text{ (4dp)}$$

Question 4. (25 points)

A scientist, that you are working with, gives you the following data

x	10.6	9.8	8.1	9.3	8.8	11.5	13.0	10.8	10.3	7.7
y	21.0	23.3	16.2	18.6	17.0	19.9	25.9	24.0	22.9	17.3

where each x and y are a pair of measurements taken on the same individual. Note that $\sum_{i=1}^n x_i = 99.9$, $\sum_{i=1}^n y_i = 206.1$ and $\sum_{i=1}^n y_i^2 = 4347.81$.

(a) Compute $\sum_{i=1}^n x_i y_i$, $\sum_{i=1}^n x_i^2$, \bar{x} and \bar{y} .

$$\sum x_i y_i = (10.6)(21.0) + (9.8)(23.3) + (8.1)(16.2) + \dots + (7.7)(17.3) = 2098.57$$

$$\sum x_i^2 = (10.6)^2 + (9.8)^2 + \dots + (7.7)^2 = 1021.21$$

$$\bar{x} = \frac{99.9}{10} = 9.99 \quad \bar{y} = \frac{206.1}{10} = 20.61$$

(b) Compute the standard deviations s_x and s_y .

$$s_x = \sqrt{\frac{1021.21 - 10(9.99)^2}{10-1}} = \sqrt{2.5788} = 1.6059 \text{ (4dp)}$$

$$s_y = \sqrt{\frac{4347.81 - 10(20.61)^2}{10-1}} = \sqrt{11.121} = 3.3348 \text{ (4dp)}$$

(c) Compute the correlation between x and y .

$$r = \frac{1}{9} \left(\frac{1}{1.6059} \right) \left(\frac{1}{3.3348} \right) (2098.57 - 10(9.99)(20.61))$$

$$= 0.8223 \text{ (4dp)}$$

(d) Interpret your correlation. What does it say about the relationship between x and y .

4 pts

Since the correlation is 0.8223 we say that there is a strong, ^{positive} linear relationship between x and y .