NAME: MODEL SOLUTIONS

Math 124 FALL 2004: Section 6 TTh 8:10-9:35 AM
Midterm 1 **Date: Mar 17, 2005**

Instructions: Answer all questions. Show as much work as you feel reasonable. You have 75 minutes. To allow others to fully concentrate at the end please do not leave in the last 5 minutes. You should submit your page of notes with your test paper.

Question 1. (*25 points*)

Suppose that you observe the following data

6.9 7.0 7.2 7.3 7.6 7.8 8.5 8.5 8.7 9.0 9.7 10.2 11.6

(a) Compute the median of this data

6pts

13 observations so $\frac{13+1}{2} = 7$th observation is median

median = 8.5

(b) Calculate the IQR.

6pts

$LQ = \frac{7.2+7.3}{2} = 7.25$ $UQ = \frac{9.0+9.7}{2} = 9.35$

$IQR = 9.35 - 7.25 = 2.1$

(c) Identify the observations that are outliers using the 1.5IQR rule discussed in class. Make it clear how you identified these observations.

6pts

Outlier if $> UQ + 1.5IQR = 9.35 + 3.15 = 12.5$
or $< LQ - 1.5IQR = 7.25 - 3.15 = 4.1$

so no outliers

(d) Draw and interpret the appropriate stem and leaf plot

7pts

6	9
7	0 2 3 6 8
8	5 5 7
9	0 7
10	2
11	6

– right (positive) skewed.

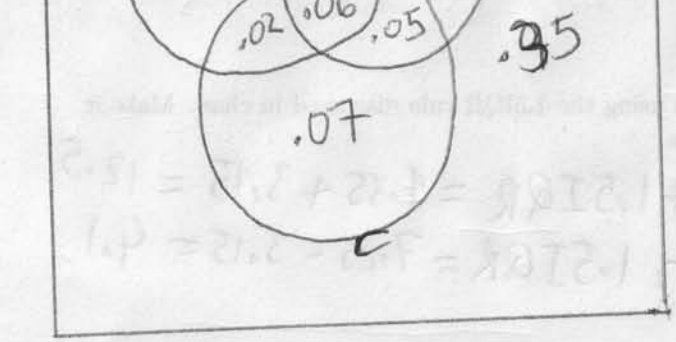
↑ represents decimal point location.

Question 2. (*25 points*)

Suppose you have three events A, B and C. Furthermore, you have the following probabilities: $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.2$, $P(A \cap C) = 0.08$, $P(B \cap C) = 0.11$, $P(A^C \cap B) = 0.28$ and $P(A \cap B \cap C) = 0.06$.

(a) Draw the appropriate Venn Diagram to represent all three events. Be sure to show all the probabilities.

9pts



(b) Are events A and B independent? How about B and C?

from Venndiagram

$P(A)P(B) = (.3)(.4) = .12 = (.06 + .06) = .12 = P(A \cap B)$

$\Rightarrow$ so A and B are independent

6pts

$P(B)P(C) = (.4)(.2) = .08 \neq .11 = P(B \cap C)$

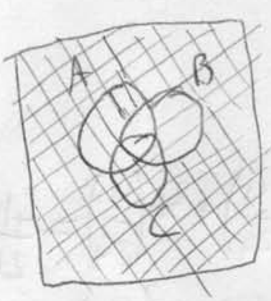
$\Rightarrow$ so B and C are dependent (not independent)

(c) What is $P(A^C \cap C)$?

$P(A^C \cap C) = .07 + .05 = .12$

5pts

(d) What is $P(A^C \cup B^C)$?



$P(A^C \cup B^C) = .16 + .23 + .02 + .05 + .07 + .35$

$= .88$

or $P(A^C \cup B^C) = P((A \cap B)^C) = 1 - P(A \cap B)$

$= 1 - .12$

$= .88$

5pts

Question 3. (*25 points*)

Imagine that you carry out a random experiment where you first roll a fair 6 sided dice, then you roll a fair 4 sided dice.

(a) Give the sample space for this experiment. Then explain what the probability of each individual outcome will be and why.

$$S = \{ (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4) \}$$

Every outcome is equally likely so probability of each individual outcome is $\frac{1}{24}$

8pts

(b) Suppose that X is "the square of the larger of the two numbers". Give the probability distribution of this random variable.

x	1	4	9	16	25	36
$P(X=x)$	$\frac{1}{24}$	$\frac{3}{24}$	$\frac{5}{24}$	$\frac{7}{24}$	$\frac{4}{24}$	$\frac{4}{24}$

8pts

(c) What are the mean and standard deviations of X?

$$\mu_X = \sum x_i p_i = 1\left(\frac{1}{24}\right) + 4\left(\frac{3}{24}\right) + 9\left(\frac{5}{24}\right) + 16\left(\frac{7}{24}\right) + 25\left(\frac{4}{24}\right) + 36\left(\frac{4}{24}\right) = \frac{414}{24} = 17.25$$

$$\sigma_X^2 = \sum x_i^2 p_i - \mu_X^2 = 1^2\left(\frac{1}{24}\right) + 4^2\left(\frac{3}{24}\right) + 9^2\left(\frac{5}{24}\right) + 16^2\left(\frac{7}{24}\right) + 25^2\left(\frac{4}{24}\right) + 36^2\left(\frac{4}{24}\right) - (17.25)^2$$

$$= \frac{4930}{24} - (17.25)^2 = 116.1875$$

9pts

$$\sigma_X = \sqrt{116.1875} = 10.7790 \text{ (4dp)}$$

Question 4. (*25 points*)

A scientist, that you are working with, gives you the following data

x	11.0	10.6	16.4	7.6	14.9	9.3	14.8	11.3	11.8	12.8
y	9.3	14.9	0.9	13.2	8.6	8.8	17.0	9.4	15.3	11.2

where each x and y are a pair of measurements taken on the same individual. Note that $\sum_{i=1}^n x_i = 120.5$, $\sum_{i=1}^n y_i = 108.6$ and $\sum_{i=1}^n y_i^2 = 1371.84$.

(a) Compute $\sum_{i=1}^n x_i y_i$, $\sum_{i=1}^n x_i^2$, $\bar{x}$ and $\bar{y}$.

7pts

$$\sum x_i y_i = (11.0)(9.3) + (10.6)(14.9) + \cdots + (12.8)(11.2) = \underline{1267.02}$$

$$\sum x_i^2 = (11.0)^2 + (10.6)^2 + \cdots + (12.8)^2 = \underline{1518.39}$$

$$\bar{x} = \frac{120.5}{10} = \underline{12.05} \qquad \bar{y} = \frac{108.6}{10} = \underline{10.86}$$

(b) Compute the standard deviations s_x and s_y.

7pts

$$s_x = \sqrt{\frac{1518.39 - 10(12.05)^2}{10-1}} = \sqrt{7.3739} = \underline{2.7155} \text{ (4dp)}$$

$$s_y = \sqrt{\frac{1371.84 - 10(10.86)^2}{10-1}} = \sqrt{21.3827} = \underline{4.6241} \text{ (4dp)}$$

(c) Compute the correlation between x and y.

$$r = \frac{1}{9}\left(\frac{1}{2.7155}\right)\left(\frac{1}{4.6211}\right)\left(1267.02 - 10(12.05)(10.86)\right)$$

7pts

$$= \underline{-0.3684} \text{ (4dp)}$$

(d) Interpret your correlation. What does it say about the relationship between x and y.

Because $r = -0.3684$ we would say that x and y are very weakly linearly related.

$\boxed{4pts}$ In otherwords, there is a very weak negative linear relationship between x and y.