MATH 124 Spring 2005

Lecture: 8
Date: Mar 3, 2005

Skills you should acquire from this lecture:

- Understand probability as describing long-term behavior.
- Knowledge of fundamental rules of probability.
- Able to define and use “complement” and “disjoint” as they apply to events and finding probabilities.
- Assigning probabilities using equally likely outcomes.

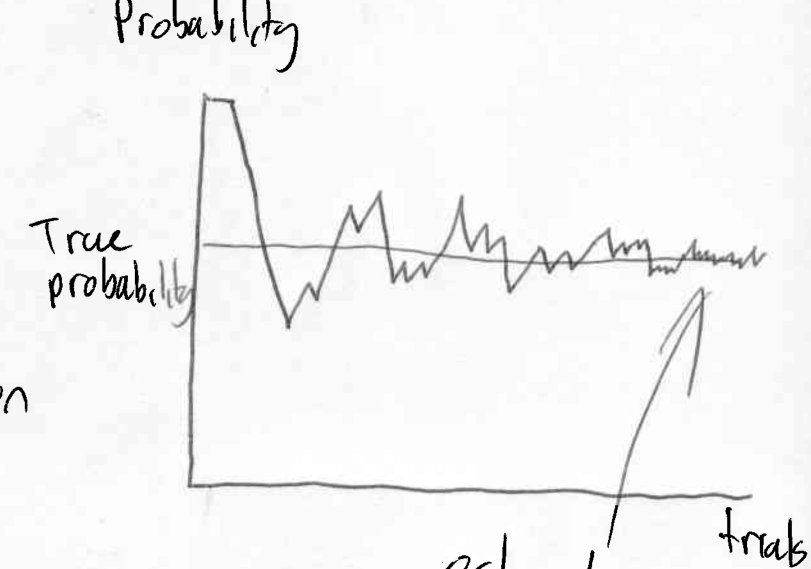
Related readings in the textbook:

- Section 4.1
- Section 4.2

Table of results for thumbtack tossing experiment from last time

Toss	#UP	P(UP)
1	1	1
2	2	1
3	2	0.66
4	2	0.5
5	3	0.6
10	5	0.5
20	13	0.65
30	20	0.66
40	26	0.65
50	31	0.62
100	61	0.61
200	117	0.585
500	299	0.598
1000	617	0.617

Visual → representation



estimate converges to true probability over long run

Can we estimate probabilities in this manner in general?
No!! Usually impossible due to many reasons.
First of all it might be impossible either physically or economically to do it. For example suppose you want to estimate the probability of an earthquake over magnitude 6 this year. Unless you have magical powers it would be impossible to make this year run over and over again.
Even if you can carry out the experiment there are still some things you need to consider:

- Repeatibility
  - Is every trial carried out in exactly the same way

repeatibility issues for thumbtack experiment
- inital state (up/down/on side/...)
- surface
- height
- toss or drop

- Independence
  - Does the outcome of any trial affect the outcome of any other trial

thumbtack experiment
- changing how you toss thumbtack depending on what happened on previous toss

Some Probability Terminology

Sample Space - set of all possible outcomes of an experiment. Denoted by S

eg single coin toss $S = \{H, T\}$

heads tails

two coin tosses

$S = \{HH, HT, TH, TT\}$

roll a standard 6 sided dice

$S = \{1, 2, 3, 4, 5, 6\}$

Political Poll: "Do you agree or disagree with the presidents social security plan?"

$S = \{$Agree, Disagree, Undecided$\}$

Toss a coin until get first heads

$S = \{H, TH, TTH, TTTH, \ldots\}$

Event - an event is an outcome or a set of outcomes of a random experiment. Events are subsets of the sample space. Usually denoted using upper case letters eg $A, B, C, \ldots$

eg Single coin toss

Event A is "get a heads"

$A = \{H\}$

two coin tosses

Event B is "two of same side"

$B = \{HH, TT\}$

roll a 6 sided dice

Event C is "Prime number showing"

$C = \{2, 3, 5\}$

Political poll

Event D is "Does not agree with president"

$D = \{\text{Disagree}, \text{Undecided}\}$

Toss coins until first heads

Event A is "heads within first 3 tosses"

$A = \{H, TH, TTH\}$

Probabilities Our notation is as follows:

$P(A)$ is read as "probability of the event A occuring"

Probability Rules

Rule 1 For any event A, $0 \leq P(A) \leq 1$
(ie probabilities are always between 0, which means impossible, and 1, which means certainty)

Rule 2 If S is the sample space then
$$P(S) = 1$$
ie the total probability for all possible outcomes is equal to 1

Rule 3 The complement of any event A is the event that A does not occur, denoted by A^c. The probability rule is
$$P(A^c) = 1 - P(A) \quad (\text{or } P(A) = 1 - P(A^c))$$

eg two coin tosses

Let $A =$ "get two heads" $= \{HH\}$

so $A^c =$ "Don't get two heads" $= \{HT, TH, TT\}$

If $P(A) = 0.25$ then $P(A^c) = 1 - P(A) = 1 - 0.25 = 0.75$

<u>Rule 4</u> Two events, A and B, are <u>disjoint</u> if they have no outcomes in common and so cannot occur simultaneously.

If A and B are disjoint then

$$P(A \text{ or } B) = P(A) + P(B)$$

eg roll a dice

Let $A =$ "Get a 1" $= \{1\}$

$B =$ "Get an even number" $= \{2, 4, 6\}$

A and B are disjoint

Suppose $P(A) = \frac{1}{6}$ $P(B) = \frac{1}{2}$

then by rule 4 $P(A \text{ or } B) = P(A) + P(B)$

$$= \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$

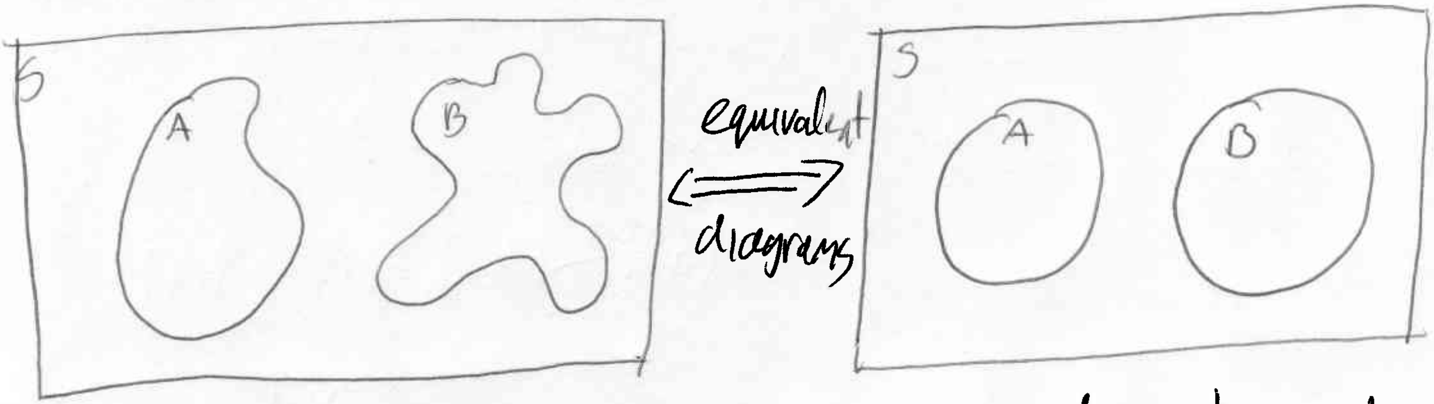
Let $A =$ "Get a prime" $= \{2, 3, 5\}$

$B =$ "Get an even" $= \{2, 4, 6\}$

A and B are not disjoint since 2 is both even and a prime

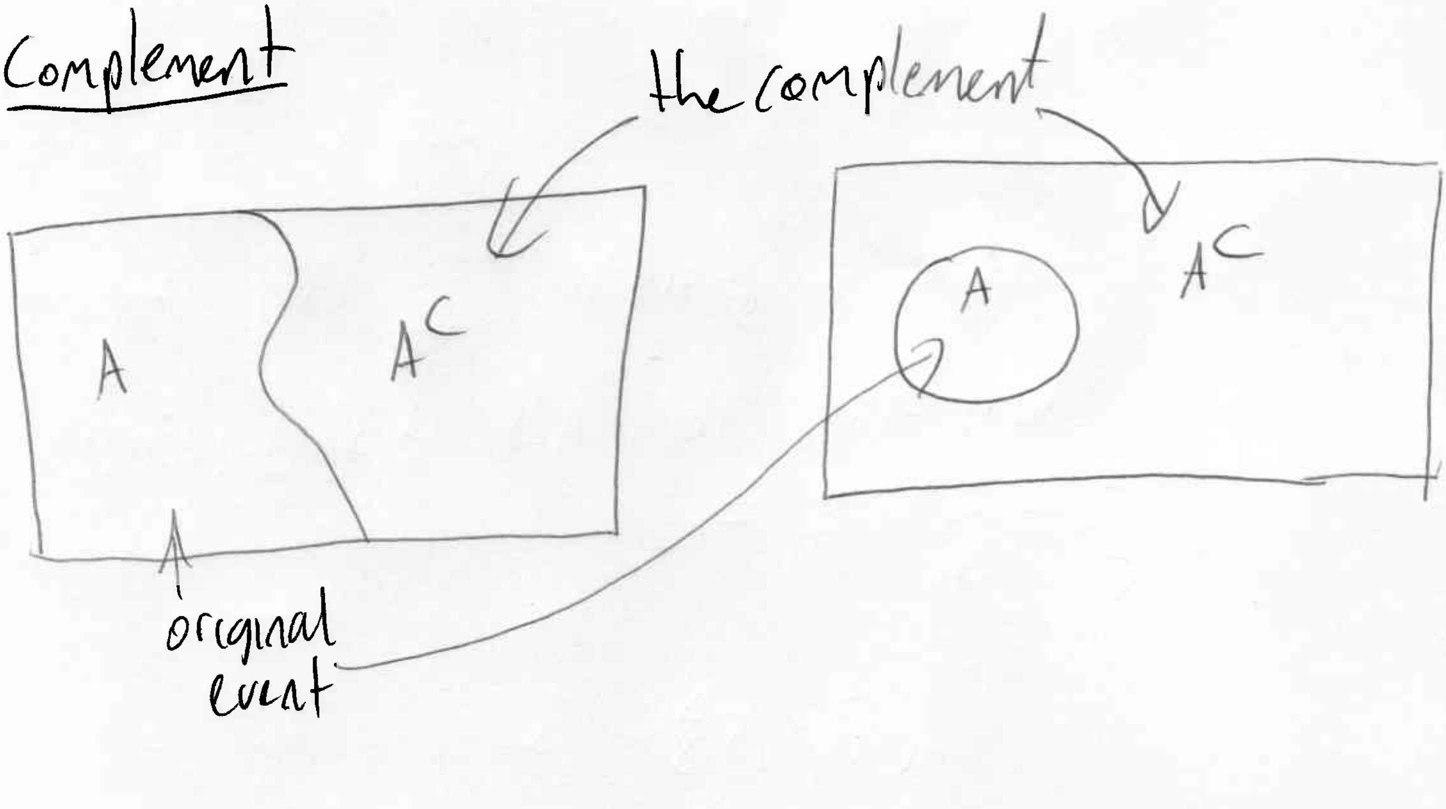
Venn diagrams

Disjoint



Since the regions for A and B do not overlap A and B are disjoint

Complement



Assigning Probabilities

when assigning probabilities need to be sure

1. Each outcome in the sample space has a probability between 0 and 1 the sum of all the probabilities should equal 1.
2. The probability of any event is the sum of the probabilities of the outcomes making up the event.

eg Suppose we have a loaded dice

Face	1	2	3	4	5	6
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$

This satisfies 1) all probabilities are between 0 and 1

and $\frac{1}{12}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{4} = \frac{1}{12}+\frac{2}{12}+\frac{2}{12}+\frac{2}{12}+\frac{2}{12}+\frac{3}{12}$

$= 1$

How about if we are concerned whether things are even or odd?

Face	Even	odd
probability	$\frac{7}{12}$	$\frac{5}{12}$

$$P(Even) = P(2) + P(4) + P(6)$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{4} = \frac{7}{12}$$

$$P(odd) = 1 - P(Even)$$
$$= 1 - \frac{7}{12}$$
$$= \frac{5}{12}$$

or

$$P(odd) = P(1) + P(3) + P(5)$$
$$= \frac{1}{12} + \frac{1}{6} + \frac{1}{6} = \frac{5}{12}$$

Equally likely Outcomes

Often it is reasonable to assume that every outcome has an equally likely chance of occuring. Some examples

- tossing a fair coin
- rolling a fair dice
- number chosen on a roullette wheel

If this is the case then if there is K possible outcomes the probability of any individual outcome is $\frac{1}{K}$

eg toss a fair coin - 2 outcomes so:

$$P(H) = P(T) = \frac{1}{2}$$

roll a fair 6 sided dice - 6 outcomes:

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

roulette wheel - 38 outcomes

$$P(i) = P(0) = P(00) = \frac{1}{38}$$

$i = 1, \ldots, 36$

The probability of an event A when every outcome is equally likely is

$$P(A) = \frac{\text{count of outcomes in } A}{\text{count of outcomes in } S}$$

ie $P(A) = \dfrac{\text{number of outcomes in } A}{K}$

eg roll a dice. Let A = "get a primenumber" = {2, 3, 5}

$$\Rightarrow P(A) = \frac{3}{6}$$ ← # of primes
← # sides of dice

roulette wheel. Let B = "get a red"

$$P(B) = \frac{18}{38} = \frac{9}{19}$$ ← # of reds

36 red/black numbers + 0 + 00