

MATH 124 Spring 2005

Lecture: 22

Date: May 12, 2005

Skills you should acquire from this lecture:

- Concept of a Standard Error (SE)
- Statistical inference for population mean when population standard deviation is not known.
- How to use the t-distribution table for confidence intervals.
- How to put bounds on a P-value using the t-distribution table.

Related readings in the textbook:

- Section 7.1

We have talked about confidence intervals and hypothesis testing for μ when σ was known. Today we address the situation when σ is unknown.

Standard error

When we estimate the standard deviation of a statistic using data the result is called the standard error of the statistic.

eg

Standard deviation

Standard error

\bar{x}

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$SE_{\bar{x}} = \frac{S}{\sqrt{n}}$$

← sample standard deviation

\hat{p}

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

eg Suppose we have a sample of data

5.1, 3.2, 4.5, 4.9, 5.7

$$\bar{x} = \frac{5.1 + 3.2 + 4.5 + 4.9 + 5.7}{5} = 4.68$$

$$s = \sqrt{\sum x_i^2} = 113$$

$$s = \sqrt{\frac{113 - 5(4.68)^2}{5-1}} = 0.9338 \text{ (4dp)}$$

so what is $SE_{\bar{x}}$?

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0.9338}{\sqrt{5}} = 0.4176 \text{ (4dp)}$$

Inference in this situation (for μ from $N(\mu, \sigma)$ with σ unknown)

Recall that $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ which has $N(0,1)$ distribution is the basis of

inference when σ/\sqrt{n} is known.

what happens if we replace σ/\sqrt{n} with the SE? (3)

Call this

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

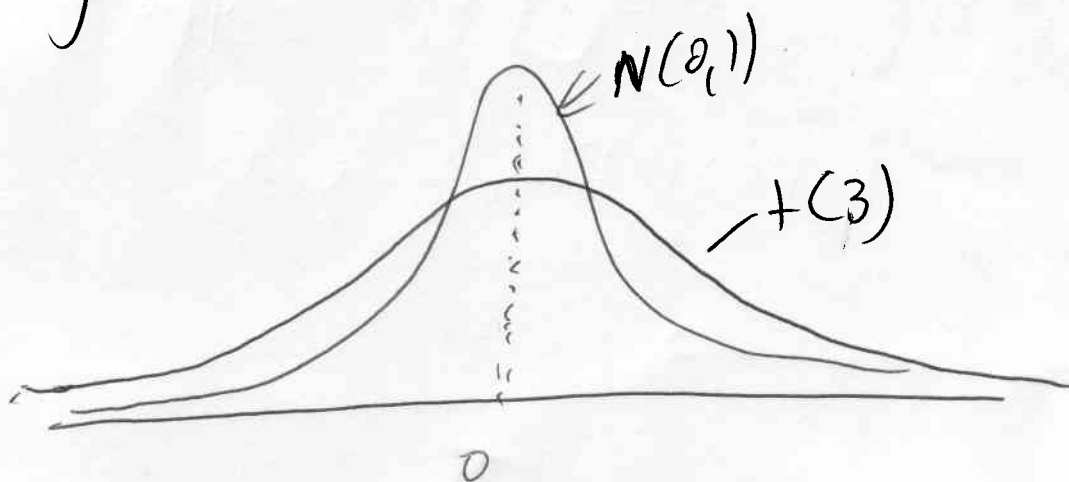
this statistic does not have the standard normal distribution. Instead it ~~is~~ is distributed according to something called the "t distribution" with $n-1$ "degrees of freedom".

what is this t-distribution?

- symmetric bell shaped curve centered at 0
- has only one parameter k which we call the degrees of freedom. ($k=1, 2, 3, \dots$)
- is more spread out than the standard normal curve
- gets less spread out as k increases
- when k approaches infinity the distribution approaches the $N(0, 1)$ curve

eg

(4)



There is a table in the back of the book that allows us to look up areas under the t -distribution.

Confidence interval for μ (when σ unknown)

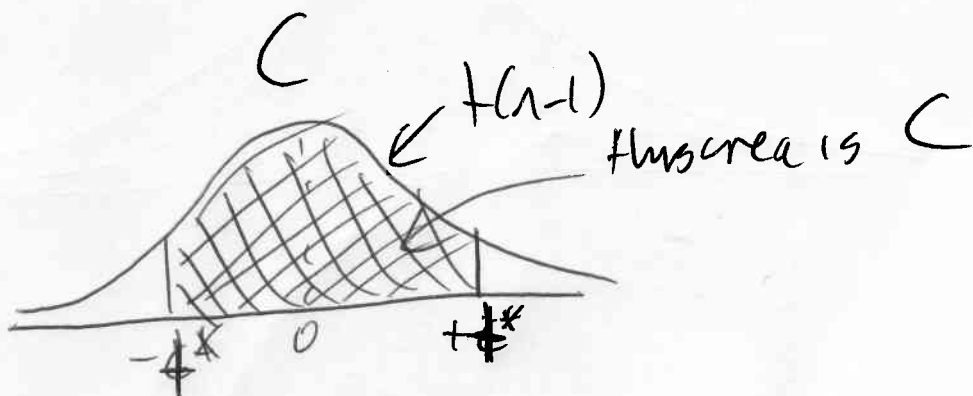
A level C confidence interval for μ is given by

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t^* is the value from the $t_{(n-1)}$ density curve with area C between $-t^*$ and t^*

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$$P(-t^* < T < t^*) = C$$



using our data from before what is a 95% CI for μ ? (Assume it is from a normal distribution)

Recall $\bar{x} = 4.68$ $SE_{\bar{x}} = .4176$ so

the 95% CI is given by

$$4.68 \pm 2.776 (.4176)$$

↑
this number is from t distribution

$$4.68 \pm 1.1592$$

so 95% CI for μ is
 $(3.52, 5.84)$.

Hypothesis tests for μ (when σ is unknown)

We proceed here in much the same way as we did when we knew σ the only difference is in how we compute the test statistic and which distribution we use

$$H_0: \mu = \mu_0$$

$$H_A: \mu \neq \mu_0$$

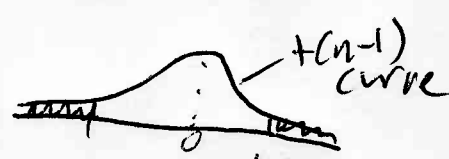
test statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

has t distribution with $n-1$ degrees of freedom

P-values come from the t-distribution table.

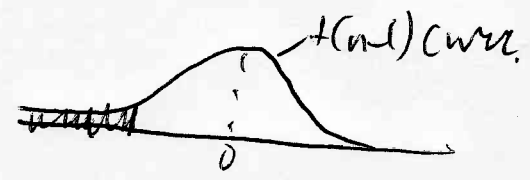
Alternative	P-value
$H_A: \mu \neq \mu_0$	$2P(T > t)$



$H_A: \mu > \mu_0$	$P(T > t)$
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$H_A: \mu < \mu_0$	$P(T < t)$
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Example

Suppose we want to test whether or not the population from which we sampled our data has mean of 5 then

$$H_0: \mu = 5$$

$$H_A: \mu \neq 5$$

The test statistic would be

$$t = \frac{4.68 - 5}{.9338/\sqrt{5}}$$

which has t-distribution with $5-1=4$ df

$$= -.7663 \text{ (4dp)}$$

$$P\text{value} = 2 P(T > |-.7663|)$$

$$= 2 P(T > .7663)$$

Since we cannot look this up exactly instead we put bounds on it. From t table df=4 line we find

$$.741 < .7663 < .941$$

$$.25 > P(T > .7663) > .20$$

↳ converting into upper tail probabilities

$$.5 > 2 P(T > .7663) > .4$$

↳ The p-value we want

Since the p-value is between .4 and .5 we have no evidence to say that the mean is not 5.