MATH 124 Spring 2005

Lecture: 20 A
Date: May 5, 2005

Skills you should acquire from this lecture:

- Example problems related to hypothesis testing for the population mean with sigma (standard deviation known).
- Problems 6.46, 6.47, 6.45, 6.63, 6.98, 6.101

As an extension of this material you might consider reading section 6.3 of the book. A few comments are at the end of this document

Problem 6.46

$H_0: \mu = 120$

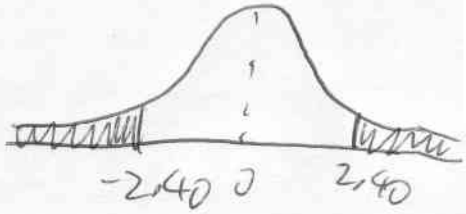
$H_A: \mu \neq 120$

test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$\bar{x} = 123.8 \quad \sigma = 10 \quad n = 40$

So $z = \dfrac{123.8 - 120}{10/\sqrt{40}} = 2.40$



P-value is $2P(Z > 2.40)$

$= 2(1 - P(Z < 2.40))$

$= 2(1 - .9918)$

$= 2(.0082)$

$= 0.0164$

Since the p-value is small we have evidence to show that the null hypothesis is not true. ie we have evidence that μ differs from 120. The conclusion is still correct even if the distribution of corn yields differs from normal, because the CLT provides that $\bar{x}$ has approximately normal distribution as n increases. With $n=40$ the approximation could be expected to be reasonable.

Problem 6.47

μ = "mean ACT score for students who take prep course"

(a) $H_0: \mu \leq 20$ (ie no improvement)

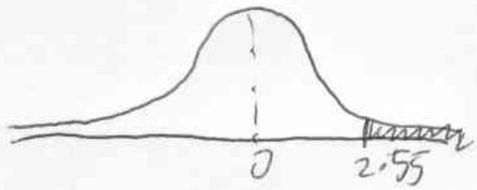
$H_A: \mu > 20$ (improvement).

$\bar{x} = 22.1$, $n = 53$, $\sigma = 6$

test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{22.1 - 20}{6/\sqrt{53}} = \frac{2.1}{(6/\sqrt{53})} = 2.5480$$

$$\text{P-value} = P(Z > 2.55)$$



$$= 1 - P(Z < 2.55)$$
$$= (1 - .9946)$$
$$= .0054$$

This is strong evidence to reject H_0 in favor of H_A ie we have good evidence that the mean score is higher than 20

b) The study should have randomly assigned half the students to get the special preparation course and half to get the standard course. then compare the mean of those who took the new course with the mean of those who did not.

Problem 6.45

a) $H_0: \mu = 115$ test statistic is

$H_A: \mu > 115$ $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$\bar{x} = 135.2$ $\sigma = 30$ $n = 20$

$$z = \frac{135.2 - 115}{30/\sqrt{20}} = \frac{20.2}{(30/\sqrt{20})} = 3.01$$

P-value $= P(Z > 3.01)$

$= 1 - P(Z < 3.01)$

$= 1 - .9987$

$= .0013$ so we would reject H_0 and accept H_A. That is we conclude that $\mu > 115$.

b) We assume a SRS (to avoid potential bias problems) and normality of the scores. The normality assumption is less important because of the CLT.

Problem 6.63

First compute $\bar{X}$.

$$\bar{X} = 104.13$$

(a) Since $\bar{X} =$, $\sigma = 9$, $n=12$

the 95% CI for μ is

$$104.13 \pm 1.96 \frac{9}{\sqrt{12}}$$

$\Rightarrow 104.13 \pm 5.09$

$\Rightarrow$ the 95% CI for μ is the interval

$(99.04 , 109.23)$

(b) Two ways to do this.

First way (normal procedure)

$H_0: \mu = 105$

$H_A: \mu \neq 105$

test statistic $z = \dfrac{104.13 - 105}{9/\sqrt{12}} = -0.33$

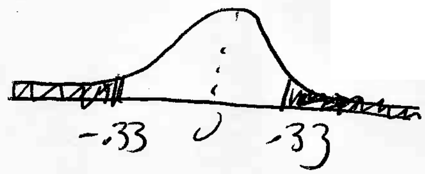
P-value is $2P(Z > |-0.33|)$

$= 2(1 - P(Z < 0.33))$

$= 2(1 - .6293)$

$= 2(.3707)$

$= .7414$ so cannot reject H_0.



<u>Second way</u> (using CI)

Is 105 in interval? Yes, then this means that a 5% level of significance test would not reject H_0. (If 105 was not in the interval then H_0 would be rejected at 5% level of significance).

Additional Online only examples

Problem 6.98

$\bar{X} = 145 \quad n = 15 \quad \sigma = 8$

(a) A 90% CI for μ (the mean cellulose content) is given by

$$145 \pm 1.645 \frac{8}{\sqrt{15}}$$

$$\Rightarrow 145 \pm 3.40$$

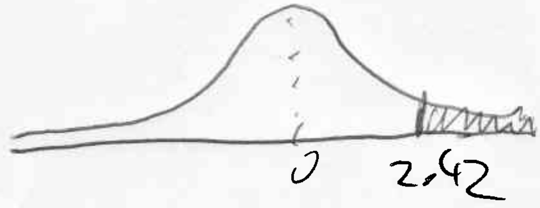
The 95% CI for the mean cellulose level is $(141.60, 148.40)$

(b) $H_0: \mu \leq 140$

$H_A: \mu > 140$

test statistic $z = \dfrac{145-140}{8/\sqrt{15}} = 2.42$

$P_{value} = P(Z > 2.42)$



$= 1 - P(Z < 2.42)$

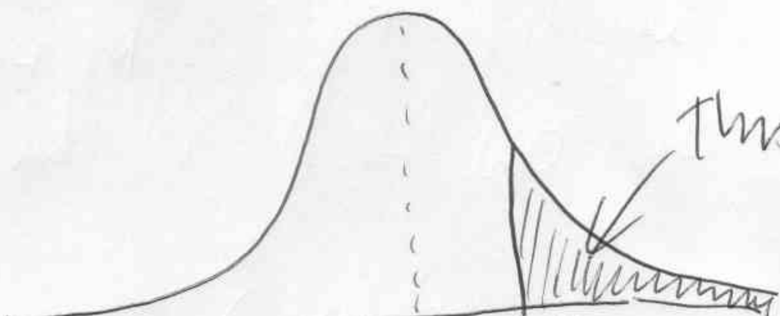
$= 1 - .9922$

$= .0078$. Since $.0078 < .05$ reject H_0 in favour of H_A. ie conclude that mean cellulose level is higher than 140.

(C) These methods assume that the data is a SRS and that the distribution of cellulose content is normally distributed.

Problem 6.101

(a)



$$Z = \frac{6.9 - 0}{55/\sqrt{104}} = 1.28$$

$$P\text{ value} = P(Z > 1.28)$$

$$= 1 - P(Z < 1.28)$$
$$= 1 - .8997$$
$$= .1003$$

(C) because the p-value is larger than $\alpha = .05$ we conclude that the null hypothesis can not be rejected. ie we have no evidence to show that the mean CEO compensation increased.

Again read Section 6.3

Key point of 6.3 is that tests aren't magical. They depend on assumptions (which may or not be true) and if you are not careful can be manipulated.