

MATH 124 Spring 2005

Lecture: 20

Date: May 3, 2005

Skills you should acquire from this lecture:

- Introduction to hypothesis testing
- Concepts of Null and Alternative hypotheses, test statistics, P-values, Making conclusions
- Carrying out hypothesis tests about the population mean when sigma is known.

Related readings in the textbook:

- Section 6.2

Today we address the topic of Hypothesis Testing.

A Hypothesis Test is a formal method of testing a claim about the population parameter (or model) based upon a sample of data.

Components of a test

1. Null hypothesis denoted by H_0
2. Alternative hypothesis denoted by H_A
3. Test statistic
4. P value or level of significance
5. Conclusion.

An Hypothesis test is designed to assess the strength of evidence against the null hypothesis.

The null hypothesis is usually a statement of "no difference" or "no effect"

The alternative ^{hypothesis} is what we hope ^{to prove} is true. It should be the opposite to the null hypothesis. ②

Note H_0 and H_A should always be stated in terms of the population parameter (not the sample statistic)

Examples

Null

Alternative

1) $H_0: \mu = 123$

$H_A: \mu \neq 123$

} sometimes called two-sided alternative

2) $H_0: \mu \leq 123$

$H_A: \mu > 123$

} sometimes called one-sided alternative

3) $H_0: \mu \geq 123$

$H_A: \mu < 123$

Test statistic

The test statistic is a quantity we compute based upon our sample data. We use it to judge whether the null or alternative hypothesis is more likely.

P-values

The p-value is the ~~the~~ probability, if we assume that H_0 is true, that the test statistic would take the value observed or something more extreme.

In practise this means the smaller the P-value the more evidence we have against H_0 .

Some example P-values

<u>P-value</u>	<u>Evidence against H_0</u>
$< .1$	very weak evidence
$< .05$	weak evidence
$< .01$	evidence
$< .001$	strong evidence
$< .0005$	very strong evidence.

You could think of it this way

H_0 : not guilty

H_A : guilty

then the smaller you make α the less likely it is that you

convict an innocent person

Drawing a conclusion

~~mark~~

IF the p-value is smaller than some value α we say that the data are statistically significant at level α

For this class unless otherwise stated assume that you are using a 5% level of significance (i.e. $\alpha = 0.05$).

So if $p\text{-value} \leq 0.05$ say "I reject H_0 in favor of H_A "

if $p\text{-value} > 0.05$ say "I cannot reject H_0 "

Testing for population mean (assuming σ is known)

Typical null hypothesis

$H_0: \mu = \mu_0$ (μ_0 is a constant chosen before test)

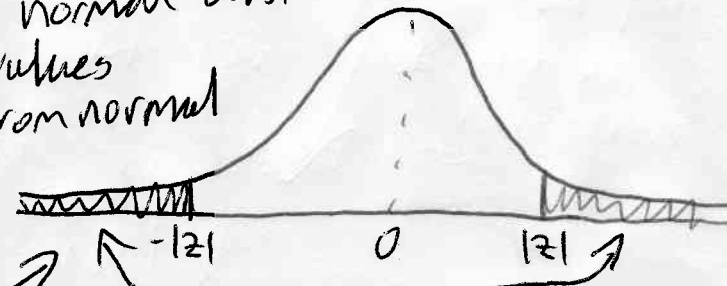
$H_A: \mu \neq \mu_0$

Test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Assuming that H_0 is true this statistic has normal distribution and

So p-values come from normal



p-value is $2P(Z > |z|)$ i.e. this area

(5)

If the alternative changes then so does the ~~new~~ region under the curve used to compute the p-value. The test statistic remains the same. "y

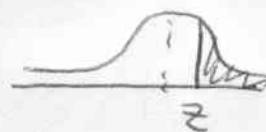
if $H_0: \mu > \mu_0$ $H_A: \mu < \mu_0$

p-value is $P(Z < z)$



if $H_0: \mu \leq \mu_0$ $H_A: \mu > \mu_0$

p-value is $P(Z > z)$



Example

Suppose that a researcher is interested in discovering whether students at SFSU are smarter than the general populace. The researcher uses IQ scores to assess whether people are smarter or not. In the general population the mean IQ score is 100 and the standard deviation is 15.

(6)

The researcher takes a SRS of size 25 from all SFSU students and finds a sample mean IQ score of 107. Is this evidence to conclude that SFSU students are smarter than average?

To answer this problem use a hypothesis test

Step 1 State the null and alternative hypothesis

$\mu \equiv$ "mean IQ score for all SFSU students"

$$H_0: \mu \leq 100$$

↑
this means

"not smarter than
general population"

$$H_A: \mu > 100$$

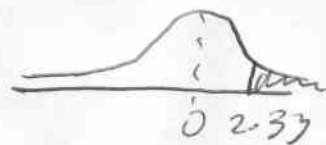
↑
"smarter than
general population"

Step 2 Compute value of test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{107 - 100}{15 / \sqrt{25}} = \frac{7}{3} = 2.33$$

Step 3 Compute the P-value

$$\begin{aligned} P\text{-value} &= P(Z > z) = P(Z > 2.33) \\ &= 1 - P(Z < 2.33) \\ &= 1 - .9901 \\ &= .0099 \end{aligned}$$



Step 4 Make a conclusion

Since $P\text{-value} = .0099 < .05$ we

may reject H_0 and accept H_A . In other words we have statistically significant evidence that SFSA students are more intelligent than the general population

(note that all figures in this example have been invented, it does not represent real life data)