MATH 124 Spring 2005

Lecture: 19 - Additional
Date: Apr 26, 2005

Discussion:

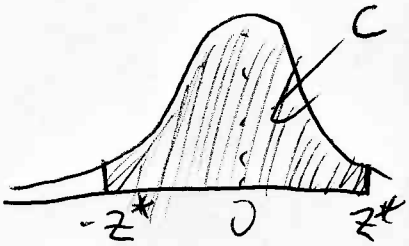
- This document includes additional worked solutions for confidence intervals for the mean when sigma is known. As general advice, you should try to work through the problems yourself before looking at the solution. This will help you learn the material for the final exam.
- Problems discussed: 6.11, 6.93, 6.3, 6.7

Problem 6.11

Recall formula for CI for μ is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$\bar{x}$ ← sample mean; σ ← population sd; n ← sample size

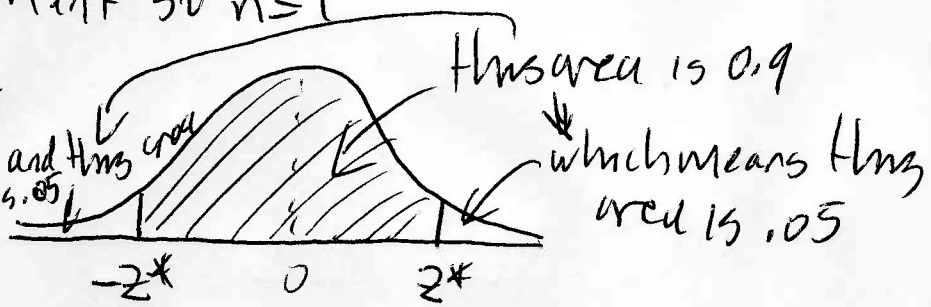


number such that $P(-z^* < Z < z^*) = C$

In this problem μ = "mean potassium level for Julie"

(a) single measurement so $n = 1$

$\bar{x} = x = 3.4$



what is z^*?

look in the table for $P(Z < -z^*) = .05$

or look in table for $P(Z < z^*) = .95$

in the table the entries .9495 and .9505 are closest to .95. These correspond to $z^* = 1.64$ and $z^* = 1.65$, since .95 is half way between .9495 and .9505 take the average of these which is $z^* = 1.645$

So 90% CI for μ is

$$3.4 \pm (1.645)\frac{(.2)}{\sqrt{1}}$$

$$\Rightarrow 3.4 \pm .3290$$

So 90% CI for mean potassium level is

$$(3.0710,\ 3.7290)$$

(b) mean of 4 measurements so $n=4$

$$\bar{x} = 3.4,\ z^* = 1.645$$

So 90% CI for μ is

$$3.4 \pm 1.645\frac{(.2)}{\sqrt{4}}$$

$$\Rightarrow 3.4 \pm (1.645)\frac{(.2)}{2}$$

$$\Rightarrow 3.4 \pm (1.645)(.1)$$

$$\Rightarrow 3.4 \pm .1645$$

So 90% CI for mean potassium level is

$$(3.2355,\ 3.5645)$$

Problem 6.93 In this problem μ = "mean phosphorus level"

First work out $\bar{x}$. $\bar{x} = \frac{5.6+5.1+4.6+4.8+5.7+6.4}{6}$

$= 5.3667$ (4dp)

$n = 6$, $\sigma = 0.9$

A 90% CI for μ is

$$5.3667 \pm 1.645\left(\frac{0.9}{\sqrt{6}}\right)$$

$$\Rightarrow 5.3667 \pm 0.6062$$

So 90% CI for μ is

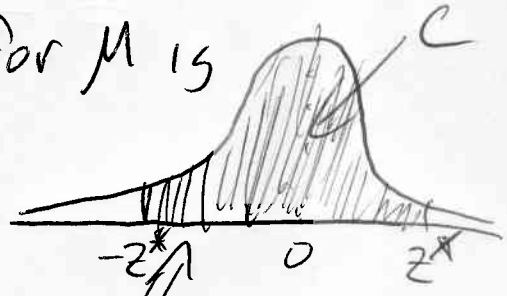
$(4.7604, 5.9729)$

Problem 6.3

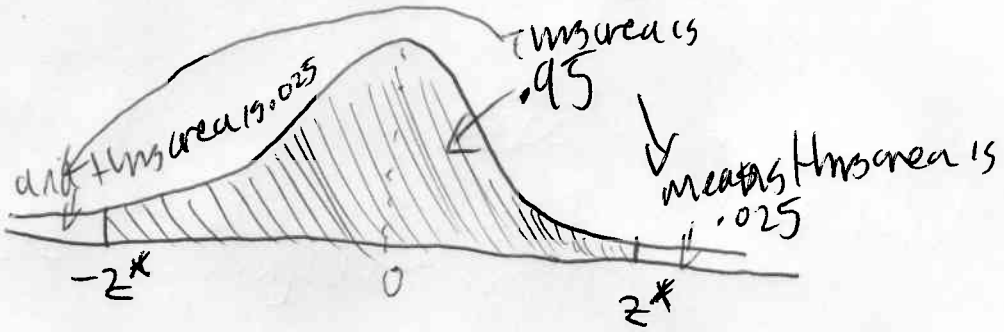
Recall that the formula for CI for μ is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$\bar{x}$: sample mean; σ: population sd; n: sample size; z^*: number such that $P(-z^* < Z < z^*) = C$



what will z^* be for a 95% CI



look in table for $P(Z < -z^*) = .025$

or look in table for $P(Z < z^*) = .975$

in table the entry .9750 corresponds to $z^* = 1.96$ (looking for .0250 corresponds with $-z^* = -1.96$)

a) $\bar{x} = 110 \quad \sigma = 40 \quad n = 25$

So 95% CI is

$$110 \pm 1.96 \frac{40}{\sqrt{25}}$$

$$\Rightarrow 110 \pm (1.96)(8)$$

$$\Rightarrow 110 \pm 15.68$$

So 95% CI for μ (the mean amount of time spent studying) is

(94.32, 125.68) minutes per week

(b) False. Our CI is for μ (the mean time spent studying) not for individual measurements.

Problem 6.7

First work out $\bar{X}$.

$$\bar{X} = 123.8$$

(a) $$123.8 \pm 1.645 \frac{10}{\sqrt{15}}$$

$$\Rightarrow 123.8 \pm 4.2474 \quad (4dp)$$

So 90% CI for μ is $(119.5526, 128.0474)$

(b) $123.8 \pm 1.96 \frac{10}{\sqrt{15}}$

$\Rightarrow 123.8 \pm 5.0607$

So 95% CI for μ is $(118.74, 128.86)$

(c) $123.8 \pm 2.57 \frac{10}{\sqrt{15}}$

$\Rightarrow 123.8 \pm 6.6357$

So 99% CI for μ is $(117.16, 130.44)$

(d) The margins of error increase as the level of confidence increases. In this example

CI Level	Margin of error
90%	4.2474
95%	5.0607
99%	6.6357

(Increasing ↓ / Increasing ↓)