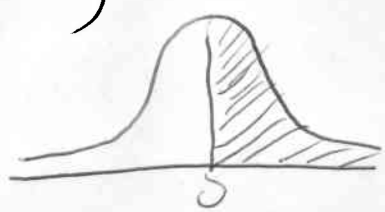
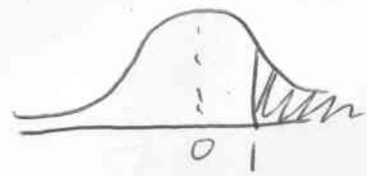
Problem 5.31

(a) we are told that $X \equiv$ "a 12 graders test score" is approximately normal with mean $\mu = 300$ and standard deviation $\sigma = 35$. so

$$P(X > 300) = P\left(\frac{X-300}{35} > \frac{300-300}{35}\right)$$
$$= P(Z > 0)$$
$$= 1 - P(Z < 0)$$
$$= 1 - .5$$
$$= .5$$



$$P(X > 335) = P\left(\frac{X-300}{35} > \frac{335-300}{35}\right)$$
$$= P(Z > 1)$$
$$= 1 - P(Z < 1)$$
$$= 1 - .8413$$
$$= .1587$$

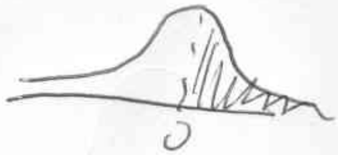


(b) Now we are interested in the mean of four 12 graders. Since each individual measurement is normal (approximately) so is the sample mean. Note that

$$\mu_{\bar{x}} = \mu = 300 \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{4}} = \frac{35}{2} = 17.5$$

$$P(\bar{x} > 300) = P\left(\frac{\bar{x}-300}{17.5} > \frac{300-300}{17.5}\right)$$

$$= P(z > 0)$$



$$= 1 - P(z < 0)$$

$$= 1 - .5$$

$$= .5$$

$$P(\bar{x} > 335) = P\left(\frac{\bar{x}-300}{17.5} > \frac{335-300}{17.5}\right)$$

$$= P(z > 2)$$

$$= 1 - P(z < 2)$$

$$= 1 - .9772$$

$$= .0228$$

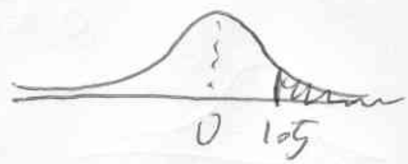
Problem 5.35

we are told that $x \equiv$ "blood glucose level for Shelia" is Normal with $\mu = 125$ $\sigma = 10$

to be diagnosed as diabetic we need $X > 140$. So

$$P(X > 140) = P\left(\frac{X-125}{10} > \frac{140-125}{10}\right)$$

$$= P(Z > 1.5)$$



$$= 1 - P(Z < 1.5)$$

$$= 1 - .9332$$

$$= .0668$$

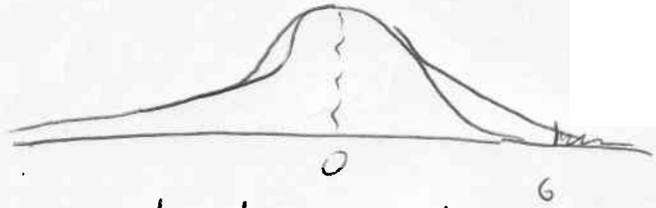
b) Now we are interested in the mean of 4 different measurements. The sample mean $\bar{X}$ will also have normal distribution, but its mean is

$$\mu_{\bar{X}} = \mu = 125$$

and standard deviation is

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{4}} = 2.5, \text{ so}$$

$$P(\bar{X} > 140) = P\left(\frac{\bar{X}-125}{2.5} > \frac{140-125}{2.5}\right)$$



$= P(Z > 6)$

$= 1 - P(Z < 6)$

$< 1 - P(Z < 3.9) = 1 - .9998$ ← largest value in table

note change of sign to "less than" rather than "equals"

$= .0002$

ie it is much less likely she will be diagnosed as diabetic if we use the mean of 4 observations rather than a single observation.

Problem 6.6

First work out $\bar{x}$.

$\bar{x} = 61.7917$ $\qquad\qquad\qquad$ $n = 24$

(a) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.5}{\sqrt{24}} = 0.9186$ (4dp)

(b) A 95% CI for μ is given by

$61.7917 \pm 1.96(0.9188)$

61.7917 ± 1.8003

so 95% CI is

$(59.99\ ,\ 63.59)$

Since this interval is completely below 65 kg we can be quite sure that mean weight is less than 65 kg.

Problem 6.8

a) what happens to $\bar{X}$ if we convert to pounds?

$$\bar{X} = 2.2(61.7917) = 135.9417$$

b) and $\sigma_{\bar{X}} = |2.2|(0.9186) = 2.0208$

c) So a 95% CI for μ in pounds

$$135.9417 \pm 1.96(2.0208)$$

$$135.9417 \pm 3.96$$

so a 95% CI for mean weight in pounds

$$(131.98, 139.90)$$