Homework 7 Solutions

3.65, 3.66, 5.5, 5.7, 5.12, 5.13

Problem 3.65

(a) No the variability will not change. The variability of the sample proportion $\left(\sqrt{\frac{p(1-p)}{n}} = \hat{\sigma}_p\right)$ depends on the sample size not the population size

(b) Yes, because now the sample sizes change from state to state. The highest populations states will have largest sample sizes and thus smallest variability. The ~~small~~ low population states will have small samples and higher variability

Problem 3.66

(a) The population is the set of all adults living in Ontario. The sample is the 61,239 adults interviewed (note that the sample is not 61,239, that is the sample size)

(b) To be close to the truth we need several things to be true including

- representative sample
  - the sample accurately reflects the population at large. Given that they took a probability sample this is probably true
- No Bias ~~due~~ to interview technique.
  - does the way in which the interview is conducted affect results. Could different interviewers have different results depending on their delivery of the questions (voice tone, facial expressions or other mannerisms). Does being interviewed skew response in one direction.

Problem 5.5

$X \equiv$ "number of Hispanics on committee"

X is Binomial $n = 15$ $p = .3$

(a) $$P(X=3) = \binom{15}{3} .3^3 (1-.3)^{15-3}$$

$$= \frac{15!}{3!\,12!} (.3^3)(.7^{12})$$

$$= .1700 \text{ (4dp)}$$

(b) $P(X \leq 3) = P(x=0) + P(X=1) + P(X=2) + P(X=3)$

$$= \binom{15}{0} .3^0 (1-.3)^{15} + \binom{15}{1} .3^1 (1-.3)^{14}$$

$$+ \binom{15}{2} .3^2 (1-.3)^{13} + \binom{15}{3} .3^3 (1-.3)^{12}$$

$$= .0047 + .0305$$

$$+ .0916 + .1700$$

$$= .2968$$

Problem 5.7

from 5.5 we learn that

X ≡ "number of Hispanics on committee of 15 members"

is Binomial with $n = 15 \quad p = 0.3$

a) For a Binomial r.v the mean is $\mu_X = np$

so

$$\mu_X = 15(.3) = 4.5$$

b) For a Binomial r.v. the standard deviation is

$$\sigma_X = \sqrt{np(1-p)} \quad \text{so}$$

$$\sigma_X = \sqrt{15(.3)(1-.3)} = 1.7748 \ (4dp)$$

c) now $p = 0.1$ so

$$\sigma_X = \sqrt{15(.1)(.9)} = 1.1619 \ (4dp)$$

with $p = .01$

$$\sigma_X = \sqrt{15(.01)(.99)} = 0.3854 \ (4dp)$$

we learn that the standard deviation gets smaller as p approaches 0

Problem 5.12

(a) We have a fixed sample size 200. A respondent can either answer that they "are committed to eating nutritious food when eating away from home" or that they "aren't committed". Since a SRS is being used it seems reasonable to assume that responses are independent and p remains same between respondents. So the binomial setting seems reasonable. If we assume the national result then $p = .4$ so Binomial(200, .4) is reasonable distribution for

$X \equiv$ "number of respondents out of 200 who 'are committed'"

(b) We want $P(75 \leq X \leq 85)$

because $np = 200(.4) = 80 > 10$

and $n(1-p) = 200(.6) = 120 > 10$

It is reasonable to use ~~a~~ normal distribution approximation to the binomial distribution. In particular,

X is approximately normal with mean $\mu = np = 200(.4) = 80$

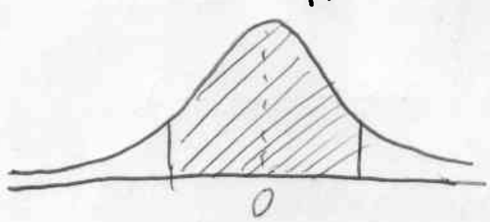
and standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{200(.4)(.6)} = \sqrt{48} = 6.9282$

So

$$P(75 \le X \le 85) \approx P\left(\frac{75-80}{6.9282} \le \frac{X-80}{6.9282} \le \frac{85-80}{6.9282}\right)$$

because of the approximation

$$= P(-0.72 \le Z \le 0.72)$$



$$= P(Z \le .72) - P(Z \le -.72)$$

$$= .7642 - .2358$$

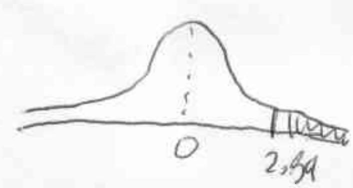
$$= .5284$$

c) What we want to know is

$$P(X \ge 100) \approx P\left(\frac{X-80}{6.9282} \ge \frac{100-80}{6.9282}\right)$$

again due to the approximation

$$= P(Z \ge 2.89)$$



$$= 1 - P(Z < 2.89)$$

$$= 1 - .9981$$

$$= .0019$$

since this is small it is not unreasonable to think p might be larger.

Problem 5.13

(a) $\hat{p} = \frac{X}{n} = \frac{62}{100} = .62$

(b) Because $np = 100(.67) = 67 > 10$
and $n(1-p) = 100(.33) = 33 > 10$

it is reasonable to use a normal approximation. In particular $\hat{p}$ has (approximately) normal distribution with mean $\mu_{\hat{p}} = p = .67$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.67(1-.67)}{100}}$
$= .0470$ (4dp)

So

$$P(\hat{p} \leq 0.62) \approx P\left(\frac{\hat{p} - .67}{.0470} \leq \frac{.62 - .67}{.0470}\right)$$

because we use normal approximation

$$= P(Z \leq -1.06)$$

$$= .1446$$

(c) Since the probability of getting 62 or smaller (ie $\hat{p} = .62$ or smaller) just by chance if $p = .67$ is not that small the survey does not support the conclusion that p is smaller at Harvard College.