NAME: SOLUTIONS

Math 124 Spring 2005: Section 6 TTh 8:10-9:25 AM
Final Exam **Date: May 26, 2005**

Instructions: Answer questions 1-8 for full credit. There is a bonus question, but you can get a perfect score without attempting it. Show as much work as you feel reasonable. You have 2 hours 30 minutes. To allow others to fully concentrate at the end please do not leave in the last 5 minutes. You should submit your pages of notes with your test paper.

Question 1. (*25 points*)

Define each of these terms in the context of sample surveys. Where appropriate explain the possible effect (of the term) on conclusions drawn from a survey.

(a) *Population* and *Sample*

5pts

population - a group of objects/units/people which you want to learn about

sample - a subset of the population for which you measure or record the variable (or variables) that you are interested in.

(b) *Simple Random Sample*

5pts

A method of random sampling where every individual in the population has an equally likely chance of being chosen for the sample. The hope would be that a SRS is more likely to give you an unbiased sample.

(c) *Stratified Random Sample*

5pts

The population is divided into groups called "strata" based on some variable(s) eg age, gender, state of residence[1], ethnicity .... Within each strata a SRS is taken. This is done to better ensure that the sample is representative of the population.

(d) *Voluntary Response*

5pts

A survey where respondants self select themselves as respondants. Eg web polls, magazine surveys. Data from voluntary response surveys is highly suspect and not likely to generalize to the population.

(e) *Question Wording*

5pts

How a question is worded can have significant effect on how a question is answered. For instance ambiguous wording could cause people who have same underlying opinion to answer differently. Using complicated terms might make it hard for the respondant to understand. Also there is potential to ask leading questions so phrasing is important.

Question 2. (*25 points*)

An education researcher at SFSU wants to investigate study habits of Freshmen students at SFSU. She gathers a list of all the Freshmen students at SFSU. Her research unit has the time and resources to deal with responses from as many as 250 students.

(a) What is the population in this study? What is the sample?

4pts

Population – Freshmen students at SFSU
Sample – 250 Freshmen students chosen for study

(b) Suggest a method she could use to select Freshmen students for her study. Explain why you recommend this method.

7pts

It would be best to use a random sampling method to avoid possible biases. Specifically a good method to use might be stratified random sampling. Divide the population into groups eg subject major, Living Arrangement etc then take SRS inside each of these groups

(c) What issues should the researcher consider when creating this questionnaire?

7pts

A primary issue which she should consider is how the questionaire is worded. Every question should be written in clear unambiguous language. Also each question should be as neutral as possible ie not suggesting one specific answer over another. Also, she should ensure that the questionnaire is not over tedious so that respondents will not resist responding due to length.

(d) The researcher has two different options on how she can carry out the study: face to face personal interviews or self-administered questionnaires which can be mailed back. Discuss potential advantages and disadvantages for each of these methods.

7pts

Method	Advantages	Disadvantages
Face to Face	- possibility of getting fuller answers	- bias due to behavior of questioner - cost/time - no anonymity
Self Admnited Questionnaire	- quicker, easier for respondent - possibly more truthful	- possibility of high level non-response - respondent can't get clarification

Question 3. (*25 points*)

Imagine that you carry out a random experiment where you first roll a fair 6 sided dice, then you roll a fair 4 sided dice.

(a) Give the sample space for this experiment. Then explain what the probability of each individual outcome will be and why.

8pts

$S = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4)\}$

since the dice are fair and independent each outcome is equally likely

$\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

(b) Suppose that X is "the sum of the two rolls". Give the probability distribution of this random variable.

8pts

x	2	3	4	5	6	7	8	9	10
$P(X=x)$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{3}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{3}{24}$	$\frac{2}{24}$	$\frac{1}{24}$

(c) What are the mean and standard deviations of X?

9pts

$$\mu_X = 2\left(\tfrac{1}{24}\right) + 3\left(\tfrac{2}{24}\right) + \cdots + 10\left(\tfrac{1}{24}\right) = 6$$

$$\Sigma x^2 p = 2^2\left(\tfrac{1}{24}\right) + 3^2\left(\tfrac{2}{24}\right) + \cdots + 10^2\left(\tfrac{1}{24}\right) = 40.1667 \text{ (4dp)}$$

$$\sigma_X^2 = \Sigma x^2 p - \mu_X^2 = 40.1667 - 36 = 4.1667$$

$$\text{So } \sigma_X = \sqrt{4.1667} = 2.0412 \text{ (4dp)}$$

Question 4. (*25 points*)

Protein is an important component of both human and animal diets. Although it is well known that grains and legumes contain large amounts of protein, it is not widely recognized that certain grasses can also provide a good source of protein. It is thought that Bermuda grass should contain 20% protein by weight. So one kilogram of Bermuda grass would contain 200g of protein. A scientist wants to verify this claim and so she gathers 75 one-kilogram samples of Bermuda grass and analyzes them for protein content. She finds the mean protein content for her sample is 180g. It is well known that the standard deviation of protein contents from one kilogram samples of grass is 80g.

(a) Give a 98% confidence interval for the mean protein content of Bermuda grass.

9pts

$$180 \pm 2.326\left(\frac{80}{\sqrt{75}}\right)$$

$$\Rightarrow 180 \pm 2.326(9.238)$$

$$\Rightarrow 180 \pm 21.487$$

$$\Rightarrow (158.513, 201.487)$$

ie a 98% CI for mean protein content of 1 kg of Bermuda grass is between 158.5g and 201.5g.

(b) Based upon your confidence interval would it be safe to say that Bermuda grass contains 20% protein by weight? Explain why.

4pts

Yes because the confidence interval computed in part a contains 200g which is 20% of 1-kg.

(c) What assumptions are you making when you compute your confidence interval? Which is most important?

3pts

1. SRS - most important
2. Normality - less important since CLT tells us that with a large enough sample size (and n=75 is not small) the sample mean $\bar{x}$ will be approximately normally distributed

Question 5. (*25 points*)

A wine importer has the opportunity to purchase a large consignment (500 bottles) of a particular 1948 wine. Because of the wines age, some of the bottles may have turned to vinegar. However the only method to determine whether the bottle is still good is to open it and drink some. Once a bottle has been opened it cannot be re-sold. The importer arranges with the seller to randomly select and open 20 bottles. Suppose that 3 of these bottles are spoiled.

(a) What is the sample estimate of the proportion of all the bottles in the shipment that are spoiled?

7pts

Let p = proportion of all bottles in shipment spoiled

$\hat{p} = \frac{3}{20} = 0.15$

(b) Give a 95% confidence interval for the proportion of bottles in the consignment that are spoiled.

$.15 \pm 1.96\sqrt{\frac{.15(1-.15)}{20}}$

10pts

$\Rightarrow .15 \pm .1564$

$\Rightarrow (0, .3064)$

(c) Suppose the importer is willing to accept the consignment if it is likely that there are no more than 100 spoiled bottles in the shipment. Based on your confidence interval should the importer accept the shipment. Explain.

8pts

based upon our confidence interval there could be between 0 and 153.2 (ie 154) spoiled bottles since many of these possibilities are above 100 the importer should not accept the shipment.

Question 6. (*25 points*)

A quality inspector at a widget factory measures the diameter of a specific part of the widget. The factory line must be shut down and re-adjusted if the diameter of this part becomes too small. Each hour the inspector takes a simple random sample of 100 widgets and if the mean diameter is below 5mm then the parts being produced are of poor quality. Suppose that for a particular hour the inspector calculates a sample mean of 4.96mm and sample standard deviation 0.2mm for the 100 widgets sampled.

(a) What is $\mathrm{SE}(\bar{x})$?

6pts

$SE(\bar{x}) = \frac{.2}{\sqrt{100}} = .02$

(b) State appropriate H_0 and H_A (null and alternative hypotheses) to test whether or not acceptable parts are being produced.

6pts

Let μ = mean diameter of widget this hour

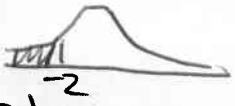
$H_0: \mu \geq 5$ (i.e machine is fine)

$H_A: \mu < 5$ (i.e parts have too small a diameter)

(c) Compute the test statistic and its P-value.

$df = 100 - 1 = 99$

$t = \frac{4.96 - 5}{.2/\sqrt{100}} = \frac{-.04}{.02} = -2$



9pts

Since H_A is $\mu < 5$ Pvalue $= P(T < -2)$

$= P(T > 2)$ (by symmetry)

using df = 80 line

$1.990 < 2 < 2.088 \Rightarrow .025 > P(T > 2) > .02$

(d) Would you recommend that the production line be shut down? Explain your answer.

4pts

Since the pvalue in part c is small we have sufficient evidence to reject H_0 in favor of H_A. So we would conclude that the mean diameter is below 5mm so factory production line should be shutdown and readjusted.

Question 7. (*25 points*)

Lead is a pollutant that can have harmful effects on humans. One method of measuring exposure to lead is to examine the lead content of human hair. One of the most common sources of lead

exposure is lead-based paint which was widely used up to the 1940's. In 1978, paint containing harmful levels of lead were banned from use on residences, furniture and toys. A research is interested in determining whether or not lead levels have changed from in the past. They gather a dataset which contains hair lead measurements in micrograms for adults who died between 1880 and 1920 and modern adults. The following table summarizes the results

Group	Size	Mean	Standard Deviation
1880-1920	30	48.5	14.5
Modern	100	26.6	12.3

You may assume that μ_1 is the mean lead level for adults in the 1880-1920 period and μ_2 is the mean lead level for modern day adults.

(a) What is the standard error of the difference between the two sample means $\bar{x}_1$ and $\bar{x}_2$?

9pts

$$SE(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{14.5^2}{30} + \frac{12.3^2}{100}} = 2.9191 \text{ (4dp)}$$

(b) Do modern adults have lower lead levels? State appropriate H_0 and H_A (null and alternative hypotheses) for answering this question.

6pts

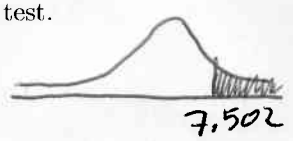
$H_0: \mu_1 - \mu_2 \leq 0$ (ie modern adults have higher or same level)

$H_A: \mu_1 - \mu_2 > 0$ (ie modern adults have lower level)

(c) Carry out this test. Be sure to state your test statistic value, its degrees of freedom and report your P-value. State in words what you conclude based upon your hypothesis test.

10pts

$$t = \frac{48.5 - 26.6}{2.9191} = 7.502 \text{ (3dp)}$$



$df = \min(29, 99) = 29$ since H_A is $\mu_1 - \mu_2 > 0$ Pvalue $= P(T > 7.502)$

from t-table $3.657 < 7.502 \Rightarrow .0005 > P(T > 7.502)$

so since Pvalue $< .0005$ very strong evidence against H_0. Conclude that modern adults have lower lead levels.

Question 8. (*25 points*)

Two insect sprays are to be compared for effectiveness at killing insects. Two rooms of equal size are sprayed. One with spray 1 and the other with spray 2. Then 100 insects are then released into each room. After 2 hours the number of dead insects are counted. In the room sprayed with spray 1 a total of 64 dead insects were found. In the other room 52 dead insects were found.

(a) Find a 96% confidence interval for the difference in proportion of insects killed be each spray.

Let p_1 = proportion of insects sprayed with spray 1 killed

p_2 = " " " " " 2 killed

$\hat{p}_1 = \frac{64}{100} = .64$ $\hat{p}_2 = \frac{52}{100} = .52$. So a 96% CI for $p_1 - p_2$

15 pts

is $.64 - .52 \pm 2.054\sqrt{\frac{.64(1-.64)}{100} + \frac{.52(1-.52)}{100}}$

$\Rightarrow .12 \pm .1423$

$\Rightarrow (-.0223, .2623)$

(b) Based upon your confidence interval or otherwise determine whether or not the two sprays differ in effectiveness.

10 pts

Since 0 is inside the CI computed in the previous step a 4% level of significance test of $H_0: p_1 - p_2 = 0$ vs $H_A: p_1 - p_2 \neq 0$ would not reject the null hypothesis therefore based on this data we cannot conclude that the sprays differ in effectiveness.