NAME: Model Solutions

Math 124 FALL 2004: Section 11 MWF 2-3
Midterm 1 **Date: Oct 1, 2004**

Instructions: Answer all questions. It is recommended that you show all work. You have 50 minutes. To allow others to fully concentrate at the end please do not leave in the last 10 minutes. You should submit your page of notes with your test paper.

Question 1. (*25 points*)

Suppose that you observe the following data

9.0 9.2 9.3 9.6 9.8 10.2 10.5 10.7 12.2 13.6

(a) Compute the median of this data

6pts

$$\text{Median} = \frac{9.8+10.2}{2} = \underline{\underline{10}}$$

(b) Calculate the IQR.

6pts

$$UQ = 10.7$$
$$LQ = 9.3$$
$$IQR = 10.7 - 9.3 = \underline{\underline{1.4}}$$

(c) Identify the observations that are outliers using the 1.5IQR rule discussed in class. Make it clear how you identified these observations.

$LQ - 1.5IQR = 9.3 - 1.5(1.4) = 9.3 - 2.1 = 7.2$

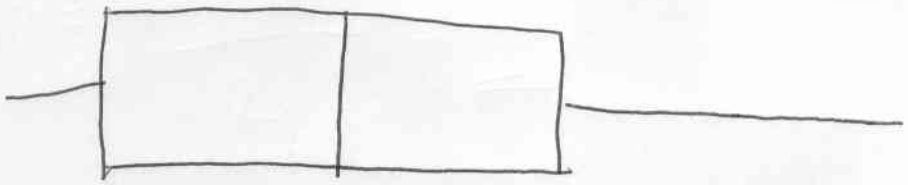
$UQ - 1.5IQR = 10.7 + 2.1 = 12.8$

6pts

So only 13.6 which is above 12.8 is an outlier

(d) Sketch the boxplot.

7pts



Question 2. (*25 points*)

Suppose that you have the following data

x	45	51	39	41	48	49	46	43	47
y	171	178	157	163	172	183	173	175	173

where each x and y are a pair of measurements taken on the same individual. Note that $\sum_{i=1}^{n} y_i = 1545$, $\sum_{i=1}^{n} y_i^2 = 265699$ and $\sum_{i=1}^{n} x_i y_i = 70416$.

(a) Compute $\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} x_i^2$, $\bar{x}$ and $\bar{y}$.

7pts

$\sum x_i = 45+51+39+41+48+49+46+43+47 = 409$

$\sum x_i^2 = 45^2+51^2+39^2+41^2+48^2+49^2+46^2+43^2+47^2 = 18707$

$\bar{x} = \frac{409}{9} = 45.44$

$\bar{y} = \frac{1545}{9} = 171.66^2$

(b) Compute the standard deviations s_x and s_y.

7pts

$$s_x = \sqrt{\frac{18707 - 9(45.44)^2}{9-1}} = \sqrt{15.03} = 3.87$$

$$s_y = \sqrt{\frac{265699 - 9(171.66)^2}{9-1}} = \sqrt{59.25} = 7.70$$

(c) Compute the correlation between x and y.

7pts

$$r = \frac{1}{n-1}\frac{1}{s_x}\frac{1}{s_y}(\Sigma x_i y_i - n\bar{x}\bar{y})$$

$$= \frac{1}{8}\frac{1}{3.87}\frac{1}{7.70}(70416 - 9(45.44)(171.66))$$

$$= 0.8560 \text{ (4dp)}$$

(d) Interpret your correlation. What does it say about the relationship between x and y.

4pts

r is positive and moderately close to 1 so we would say that there is a strong positive linear relationship between x and y

Question 3. (*25 points*)

We have a box with 8 tickets. Each ticket is labeled with a distinct number from 2 to 9. We draw two tickets from the box. What is the probability of picking a prime on the first draw {2,3,5,7} and a divisor of 6 on the second draw {2,3,6}?

(a) With replacement?

6pts

P(prime on first ∩ divisor of six on second)
= P(prime on first) P(divisor of six on second)
$= \frac{4}{8}\ \frac{3}{8} = \frac{12}{64} = \frac{3}{16}$

(b) Without replacement?

7pts

Two situations: Case 1 {draw {5,7} on first draw $\frac{2}{8}$, then draw 2,3 or 6 on second $\frac{3}{7}$
Case 2 {draw 2 or 3 on first draw $\frac{2}{8}$, then draw 6 or the remaining of 2,3 $\frac{2}{7}$

Probability
$= \frac{2}{8}\times\frac{3}{7} + \frac{2}{8}\times\frac{2}{7}$
$= \frac{10}{42} = \frac{5}{21}$

(c) Now assume that you draw only one ticket. What is the probability that the ticket is both a prime and a divisor of six?

6pts

The numbers {2,3} are both primes and divisors of 6

so probability $= \frac{2}{8} = \frac{1}{4}$

(d) Again assuming that you draw only one ticket. What is the probability that the ticket is either a prime or a divisor of six or both?

6 pts

The numbers $\{2,3,5,6,7\}$ are either prime, a divisor of 6 or both. So probability $= 5/8$

Alternatively $P(\text{prime or divisor of 6})$

$= P(\text{prime}) + P(\text{divisor of 6}) - P(\text{prime and divisor of 6})$

$= \frac{4}{8} + \frac{3}{8} - \frac{2}{8} = \frac{5}{8}$

Question 4. (*25 points*)

The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 266 days and standard deviation 16 days.

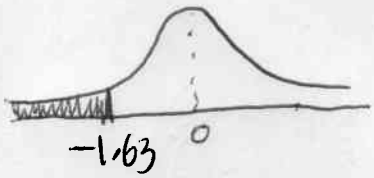
(a) What is the probability of a pregnancy lasting less than 240 days?

8 pts

$$P(X<240) = P\left(\frac{X-266}{16} < \frac{240-266}{16}\right)$$

$$= P(z < -1.63)$$

$$= .0516 \quad \text{(from table)}$$



(b) What is the probability that a pregnancy will last between 240 and 270 days?

9 pts

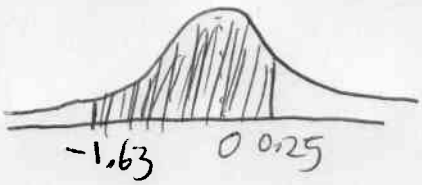
$$P(240<X<270) = P\left(\frac{240-266}{16} < X < \frac{270-266}{16}\right)$$

$$= P(-1.63 < X < 0.25)$$

$$= P(X<0.25) - P(X<-1.63)$$

$$= .5487 - .0516 \quad \text{(from table)}$$

$$= .5471$$



(c) What is the probability that a pregnancy lasts more than 275 days?

8 pts

$$P(X>275) = P\left(\frac{X-266}{16} > \frac{275-266}{16}\right)$$

$$= P(z > 0.56)$$

$$= 1 - P(z<0.56)$$

$$= 1 - .7123 = .2877 \quad \text{(from table)}$$