NAME: Model Solutions

Math 124 FALL 2004: Section 12 MWF 11-12
Midterm 1 **Date: Oct 1, 2004**

Instructions: Answer all questions. It is recommended that you show all work. You have 50 minutes. To allow others to fully concentrate at the end please do not leave in the last 10 minutes. You should submit your page of notes with your test paper.

Question 1. (*25 points*)

Suppose that you have the following data

x	12.8	12.9	12.9	13.6	14.5	14.6	15.1	17.5	19.5	20.8
y	5.5	6.2	6.3	7.0	7.8	8.3	7.1	10.0	10.8	11.0

where each x and y are a pair of measurements taken on the same individual. Note that $\sum_{i=1}^{n} y_i = 80$, $\sum_{i=1}^{n} y_i^2 = 675.16$ and $\sum_{i=1}^{n} x_i y_i = 1282.74$.

(a) Compute $\sum_{i=1}^{n} x_i$, $\sum_{i=1}^{n} x_i^2$, $\bar{x}$ and $\bar{y}$.

$$\sum x_i = 12.8+12.9+12.9+13.6+14.5+14.6+15.1+17.5+19.5+20.8$$
$$= \underline{154.2}$$

7pts

$$\sum x_i^2 = (12.8)^2+(12.9)^2+(12.9)^2+(13.6)^2+(14.5)^2+(14.6)^2+(15.1)^2+(17.5)^2$$
$$+(19.5)^2+(20.8)^2$$
$$= \underline{2452.18}$$

$$\bar{X} = \frac{154.2}{10} = \underline{15.42} \qquad \bar{y} = \frac{80}{10} = \underline{8}$$

(b) Compute the standard deviations s_x and s_y.

$$s_x = \sqrt{\frac{2452.18 - 10(15.42)^2}{10-1}} = \sqrt{8.2684} = \underline{2.8755} \ (4dp)$$

7pts

$$s_y = \sqrt{\frac{675.16 - 10(8)^2}{10-1}} = \sqrt{3.9067} = \underline{1.9765} \ (4dp)$$

(c) Compute the correlation between x and y.

$$r = \frac{1}{n-1}\frac{1}{s_x}\frac{1}{s_y}\left(\sum x_i y_i - n\bar{x}\bar{y}\right)$$

$\boxed{7pts}$

$$= \frac{1}{9}\,\frac{1}{2.8755}\,\frac{1}{1.9765}\left(1282.74 - 10(8)(15.42)\right)$$

$$= .9607 \ (4dp)$$

(d) Interpret your correlation. What does it say about the relationship between x and y.

$\boxed{4pts}$

$r = 0.9607$ is positive and near 1 so we would say that there is a strong positive linear relationship between x and y.

Question 2. (*25 points*)

Suppose that you observe the following data

9.0 9.2 9.3 9.6 9.8 10.5 10.5 10.7 12.2 13.6

(a) Compute the median of this data

$\boxed{6pts}$

$$\text{median} = \frac{9.8+10.5}{2} = 10.15$$

(b) Calculate the IQR.

$UQ = 10.7 \qquad LQ = 9.3$

6pts

$IQR = 10.7 - 9.3 = \underline{\underline{1.4}}$

(c) Identify the observations that are outliers using the 1.5IQR rule discussed in class. Make it clear how you identified these observations.

$UQ + 1.5IQR = 10.7 + 1.5(1.4) = 12.8$

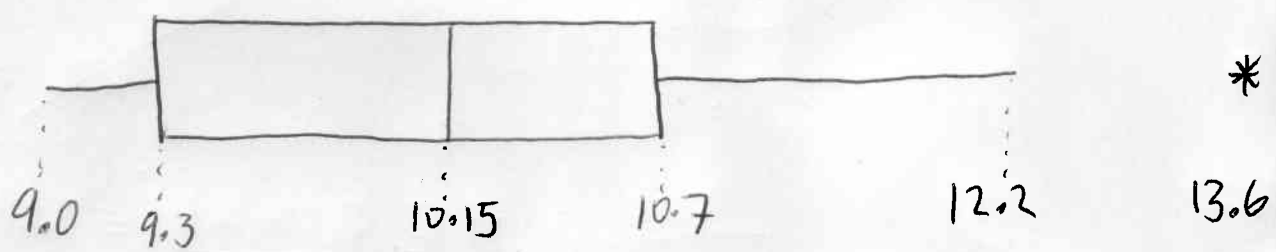
$LQ - 1.5IQR = 9.3 - 1.5(1.4) = 7.2$

6pts

Any observation above 12.8 or below 7.2 is an outlier. So only <u>13.6</u> is an outlier

(d) Sketch the boxplot.

7pts



Question 3. (*25 points*)

We have a box with 9 tickets. Each ticket is labeled with a distinct number from 1 to 9. We draw two tickets from the box. What is the probability of picking a prime on the first draw {2,3,5,7} and a divisor of 6 on the second draw {1,2,3,6}?

(a) With replacement?

P(prime on first draw) $= \frac{4}{9}$ P(divisor of 6 on second draw) $= \frac{4}{9}$

6pts

$$\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$$

(b) Without replacement? Need to get a prime and then a divisor of 6

two cases. First case: pick ~~3~~ 5 or 7 on first draw $\frac{2}{9}$

then pick 1 or 2 or 3 or 6 on second $\frac{4}{8}$

second case: pick 2 or 3 of first draw $\frac{2}{9}$

then pick 1 or 6 or remaining of 2, 3 $\frac{3}{8}$

7pts

$$\frac{2}{9} \times \frac{4}{8} + \frac{2}{9} \times \frac{3}{8} = \frac{8+6}{72} = \frac{14}{72} = \frac{7}{35}$$

(c) Now assume that you draw only one ticket. What is the probability that ~~the ticket~~ is both a prime and a divisor of six?

the outcomes {2, 3} are the only numbers that are both divisors of 6 and prime

6pts

probability $= \frac{2}{9}$

(d) Again assuming that you draw only one ticket. What is the probability that the ticket is either a prime or a divisor of six or both?

6pts

the outcomes $\{1,2,3,5,6,7\}$ are either prime or divisors of 6 or both so

probability $= \frac{6}{9} = \underline{\underline{\frac{2}{3}}}$ Alternatively $P(\text{prime}) = \frac{4}{9}$ $P(\text{divisor of } 6) = \frac{4}{9}$

$P(\text{both}) = \frac{2}{9}$

$P(\text{prime} \cup \text{divisor of six}) = \frac{4}{9} + \frac{4}{9} - \frac{2}{9} = \frac{6}{9} = \underline{\underline{\frac{2}{3}}}$

Question 4. (*25 points*)

The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 266 days and standard deviation 16 days.

(a) What is the probability of a pregnancy lasting less than 235 days?

8pts

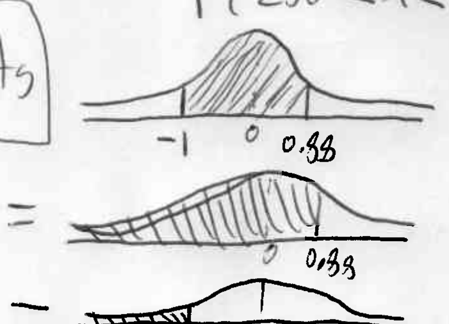
$$P(X<235) = P\left(\frac{X-266}{16} < \frac{235-266}{16}\right)$$
$$= P(Z < -1.94)$$
$$= \underline{\underline{.0262}}$$

(from table)

(b) What is the probability that a pregnancy will last between 250 and 280 days?

9pts

$$P(250<X<280) = P\left(\frac{250-266}{16} < \frac{X-266}{16} < \frac{280-266}{16}\right)$$



$$= P(-1 < Z < 0.88)$$
$$= .8106 - .1587$$
$$= \underline{\underline{0.6519}}$$

(from table)

(c) What is the probability that a pregnancy lasts more than 280 days?

8pts

$$P(X>280) = P\left(\frac{X-266}{16} > \frac{280-266}{16}\right) = P(Z>0.88)$$

$$= 1 - P(Z<0.88)$$
$$= 1 - .8106$$
$$= \underline{\underline{0.1894}}$$

(from table)