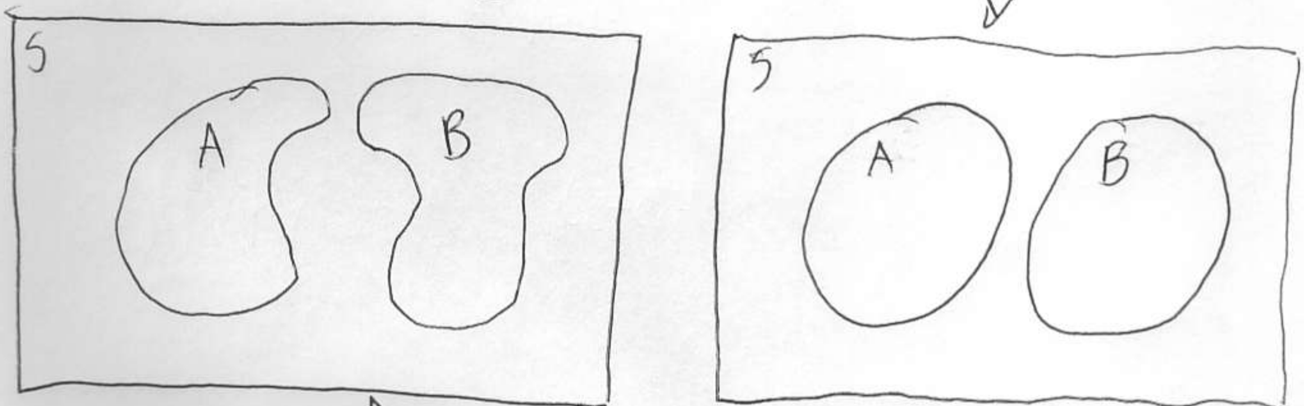
Lecture 9

Venn diagrams

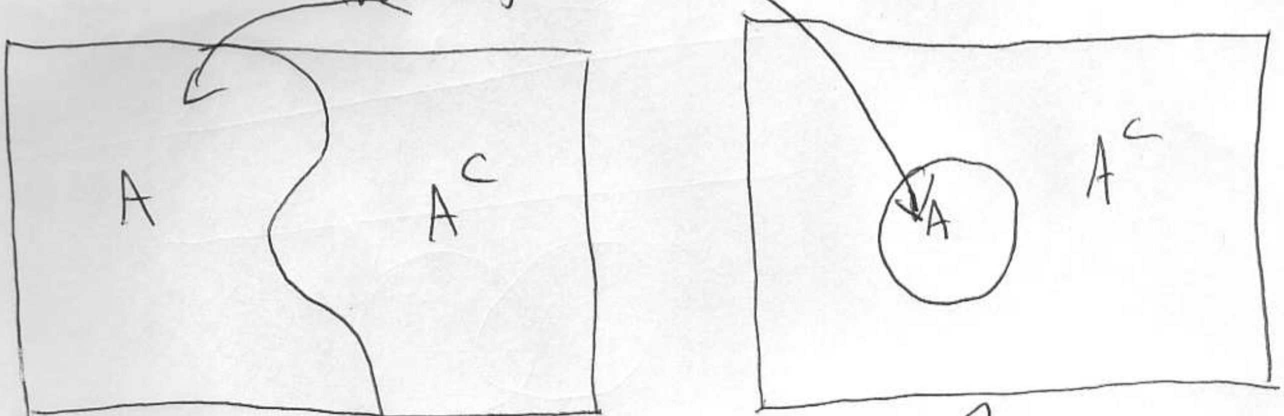
Useful for graphical representation of sample space and events

Equivalent diagrams



A and B are disjoint

The original event



The complements

Assigning Probabilities

1. Each outcome in the sample space has a probability between 0 and 1. The sum of the probabilities should equal 1.
2. The probability of any event is the sum of the probabilities of the outcomes making up the event.

Eg Suppose we have a loaded die

Face	1	2	3	4	5	6
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{4}$

All probabilities are between 0 and 1

$$\frac{1}{12}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{4}$$
$$=\frac{1}{12}+\frac{2}{12}+\frac{2}{12}+\frac{2}{12}+\frac{2}{12}+\frac{3}{12}=\underline{1}$$

$$P(\text{Even}) = P(2)+P(4)+P(6)$$
$$=\frac{1}{6}+\frac{1}{6}+\frac{1}{4}=\frac{7}{12}$$

Equally Likely Outcomes

Often it is reasonable to assume that every outcome has an equally likely chance of occuring

eg toss a fair coin
roll a fair dice
number chosen on a roulette wheel

If this is the case then if there is k possible outcomes the probability of any individual outcome is $\frac{1}{k}$

eg toss a fair coin $P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

roll a fair dice $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$

roulette wheel $P(i) = P(0) = P(00) = \frac{1}{38}$

$i = 1, \ldots, 36$

The probability of an event A when every outcome is equally likely is

$$P(A) = \frac{\text{count of outcomes in } A}{\text{count of outcomes in } S}$$

ie $P(A) = \frac{\text{number of outcomes in } A}{K}$

eg roll a dice $A =$ "Prime number" $= \{2, 3, 5\}$

$$\Rightarrow P(A) = \frac{3}{6} \quad \begin{matrix} \leftarrow \{2,3,5\} \\ \leftarrow \text{\# of sides of dice} \end{matrix}$$

$$= \frac{1}{2}$$

roulette wheel $A =$ "get a red"

$$P(A) = \frac{18}{38} = \frac{9}{19}$$

Independence

~~If~~ Two events A and B are independent if knowing that one occured does not Change the probability that the other occurs.

If A and B are independent

$$P(A \text{ and } B) = P(A)P(B)$$

eg Roll two dice

$A =$ "First roll is even"

$B =$ "Second roll is odd"

Knowing the outcome of the first event should not change the probability of the second event

$$P(A) = \frac{3}{6} = \frac{1}{2} \qquad P(B) = \frac{3}{6} = \frac{1}{2}$$

$$P(A \text{ and } B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

A = "First roll is 4, 5, 6"

B = "sum of two rolls is 7"

$$P(A) = \frac{3}{6} = \frac{1}{2} \quad \left(\frac{18}{36}\right)$$

$$\left.\begin{array}{l} P(B) = ? \\ \\ P(A \text{ and } B) = ? \end{array}\right\}$$ list all outcomes and count

Event B

	2nd roll					
First roll	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
A	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$P(B) = \frac{6}{36} = \frac{1}{6}$

$A \text{ and } B = \{(6,1), (5,2), (4,3)\}$

$P(A \text{ and } B) = \frac{3}{36} = \frac{1}{12}$

so are A and B independent

$P(A)P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} = P(A \text{ and } B)$

therefore A and B are independent.

Venn diagram for "A and B"