Lecture 24

Another problem solving session concentrating on CI and HT for μ problems. As previously the schedule is

15 minutes 1st problem
10 minutes discuss solutions
15 minutes 2nd problem
10 minutes discuss solutions.

As normal you should work in groups if you wish.

MWF 11-12		MWF 2-3
6.93, 6.11	discussed in class	6.7, 6.3
6.46, 6.47		6.45, 6.63 part (b) (part (a) is online)

Bonus online examples (ie not discussed in class)
6.98, 6.101

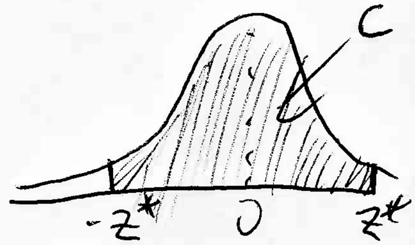
Also highly recommended that you read Section 6.3!! (some brief comments are at the end of this document)

MWF 11-12 Examples

Problem 6.11

Recall formula for CI for μ is

$$\underbrace{\bar{X}}_{\text{sample mean}} \pm z^* \frac{\sigma \leftarrow \text{population sd}}{\sqrt{n} \leftarrow \text{sample size}}$$



z^*: number such that $P(-z^* < Z < z^*) = C$

In this problem μ = "mean potassium level for Julie"

(a) single measurement so $n = 1$

$\bar{x} = x = 3.4$

what is z^*?

look in the table for $P(Z < -z^*) = .05$

or look in table for $P(Z < z^*) = .95$

in the table the entries .9495 and .9505 are closest to .95. These correspond to $z^* = 1.64$ and $z^* = 1.65$, since .95 is half way between ~~.9495~~ and ~~.9505~~ take the average of these which is $z^* = 1.645$

So 90% CI for μ is

$$3.4 \pm (1.645)\frac{(.2)}{\sqrt{1}}$$

$\Rightarrow 3.4 \pm .3290$

So 90% CI for mean potassium level is

$(3.0710, 3.7290)$

(b) mean of 4 measurements so $n=4$

$\bar{x} = 3.4$, $z^* = 1.645$

So 90% CI for μ is

$$3.4 \pm 1.645 \frac{(.2)}{\sqrt{4}}$$

$\Rightarrow 3.4 \pm (1.645)\frac{(.2)}{2}$

$\Rightarrow 3.4 \pm (1.645)(.1)$

$\Rightarrow 3.4 \pm .1645$

So 90% CI for mean potassium level is

$(3.2355, 3.5645)$

Problem 6.93 In this problem μ = "mean phosphorus level"

First work out $\bar{x}$. $\bar{x} = \dfrac{5.6+5.1+4.6+4.8+5.7+6.4}{6}$

$= 5.3667$ (4dp)

$n=6$, $\sigma = 0.9$

A 90% CI for μ is

$$5.3667 \pm 1.645\left(\frac{0.9}{\sqrt{6}}\right)$$

$$\Rightarrow 5.3667 \pm 0.6062$$

So 90% CI for μ is

$(4.7604, 5.9729)$

Problem 6.46

$H_0: \mu = 120$

$H_A: \mu \neq 120$

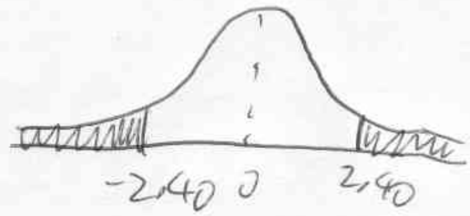
test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$\bar{x} = 123.8$ $\sigma = 10$ $n = 40$

So $z = \dfrac{123.8 - 120}{10/\sqrt{40}} = 2.40$

P-value is $2P(Z > 2.40)$



$$= 2(1 - P(Z < 2.40))$$
$$= 2(1 - .9918)$$
$$= 2(.0082)$$
$$= 0.0164$$

Since the p-value is small we have evidence to show that the null hypothesis is not true. ie we have evidence that μ differs from 120. The conclusion is still correct even if the distribution of corn yields differs from normal, because the CLT provides that $\bar{x}$ has approximately normal distribution as n increases. With $n=40$ the approximation could be expected to be reasonable.

<u>Problem 6.47</u> μ = "mean ~~ACT~~ score for students who take prep course"

(a) $H_0: \mu \leq 20$ (ie no improvement

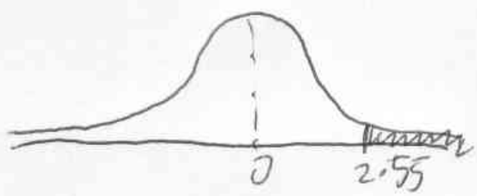
$H_A: \mu > 20$ (improvement).

$\bar{x} = 22.1$, $n=53$, $\sigma=6$

test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{22.1 - 20}{6/\sqrt{53}} = \frac{2.1}{(6/\sqrt{53})} = 2.5480$$

$$\begin{aligned} \text{P-value} &= P(Z > 2.55) \\ &= 1 - P(Z < 2.55) \\ &= (1 - .9946) \\ &= .0054 \end{aligned}$$



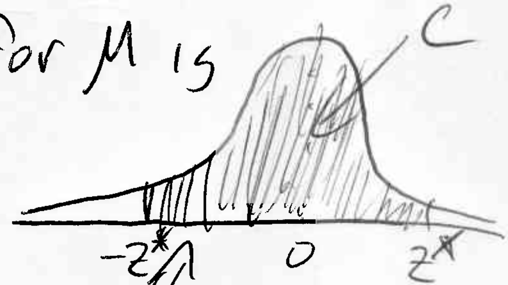
This is strong evidence to reject H_0 in favor of H_A
ie we have good evidence that the mean score is higher than 20

b) The study should have randomly assigned half the students to get the special preparation course and half to get the standard course.
then compare the mean of those who took the new course with the mean of those who did not.

MWF 2–3 Examples

Problem 6.3

Recall that the formula for CI for μ is



$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}$$

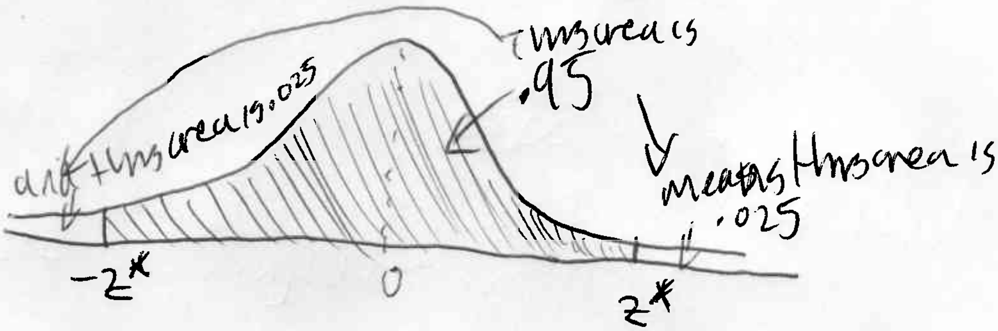
σ ← population sd

$\sqrt{n}$ ← samplesize

$\bar{X}$: sample mean

z^*: number such that $P(-z^* < Z < z^*) = C$

what will z^* be for a 95% CI



look in table for $P(Z < -z^*) = .025$

or look in table for $P(Z < z^*) = .975$

in table the entry .9750 corresponds to $z^* = 1.96$ (looking for .0250 corresponds with $-z^* = -1.96$)

a) $\bar{X} = 110 \quad \sigma = 40 \quad n = 25$

So 95% CI is

$$110 \pm 1.96 \frac{40}{\sqrt{25}}$$

$$\Rightarrow 110 \pm (1.96)(8)$$

$$\Rightarrow 110 \pm 15.68$$

So 95% CI for μ (the mean amount of time spent studying) is

$(94.32, 125.68)$ minutes per week

(b) False. Our CI is for μ (the mean time spent studying) not for individual measurements.

Problem 6.7

First work out $\bar{X}$.

$$\bar{X} = 123.8$$

(a) $123.8 \pm 1.645 \frac{10}{\sqrt{15}}$

$$\Rightarrow 123.8 \pm 4.2474 \quad (4dp)$$

So 90% CI for μ is $(119.5526, 128.0474)$

(b) $123.8 \pm 1.96 \dfrac{10}{\sqrt{15}}$

$\Rightarrow 123.8 \pm 5.0607$

So 95% CI for μ is $(118.74, 128.86)$

(c) $123.8 \pm 2.57 \dfrac{10}{\sqrt{15}}$

$\Rightarrow 123.8 \pm 6.6357$

So 99% CI for μ is $(117.16, 130.44)$

(d) The margins of error increase as the level of confidence increases. In this example

CI Level	Margin of error
90%	4.2474
95%	5.0607
99%	6.6357

increasing

increasing

Problem 6.45

a) $H_0: \mu = 115$ test statistic is

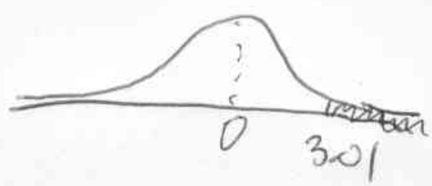
$H_A: \mu > 115$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

$\bar{x} = 135.2 \quad \sigma = 30 \quad n = 20$

$$z = \frac{135.2 - 115}{30/\sqrt{20}} = \frac{20.2}{(30/\sqrt{20})} = 3.01$$

P-value = $P(Z > 3.01)$



$= 1 - P(Z < 3.01)$

$= 1 - .9987$

$= .0013$ so we would reject H_0 and accept H_A. That is we conclude that $\mu > 115$.

b) We assume a SRS (to avoid potential bias problems) and normality of the scores. The normality assumption is less important because of the CLT.

Problem 6.63

First compute $\bar{X}$.

$$\bar{X} = 104.13$$

(a) Since $\bar{X} =$, $\sigma = 9$, $n = 12$

the 95% CI for μ is

$$104.13 \pm 1.96 \frac{9}{\sqrt{12}}$$

$\Rightarrow 104.13 \pm 5.09$

$\Rightarrow$ the 95% CI for μ is the interval

$(99.04, 109.23)$

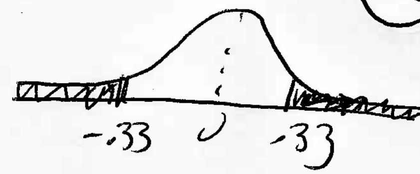
(b) Two ways to do this.

First way (normal procedure)

H_0: $\mu = 105$

H_A: $\mu \neq 105$

test statistic $z = \frac{104.13 - 105}{9/\sqrt{12}} = -0.33$



P-value is $2P(Z > |-0.33|)$

$$= 2(1 - P(Z < 0.33))$$
$$= 2(1 - .6293)$$
$$= 2(.3707)$$
$$= .7414$$ so cannot reject H_0.

<u>Second way</u> (using CI)

<u>Is</u> 105 in interval? Yes, then this means that a 5% level of significance test would not reject H_0. (If 105 was not in the interval then H_0 would be rejected at 5% level of significance).

Additional Online only examples

Problem 6.98

$\bar{X} = 145 \qquad n = 15 \qquad \sigma = 8$

(a) A 90% CI for μ (the mean cellulose content) is given by

$$145 \pm 1.645 \frac{8}{\sqrt{15}}$$

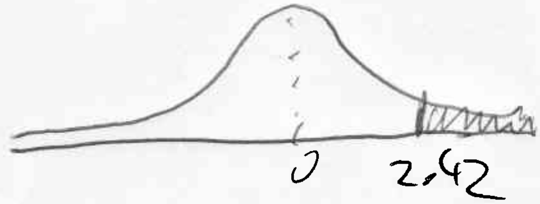
$$\Rightarrow 145 \pm 3.40$$

The 95% CI for the mean cellulose level is (141.60, 148.40)

(b) $H_0: \mu \leq 140$

$H_A: \mu > 140$

test statistic $z = \dfrac{145 - 140}{8/\sqrt{15}} = 2.42$



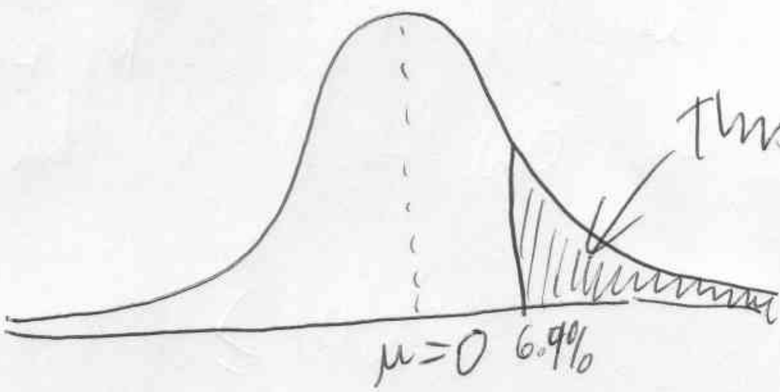
$$\text{Pvalue} = P(Z > 2.42)$$
$$= 1 - P(Z < 2.42)$$
$$= 1 - .9922$$
$$= .0078$$

Since $.0078 < .05$ reject H_0 in favour of H_A. ie conclude that mean cellulose level is higher than 140.

(C) These methods assume that the data is a SRS and that the distribution of cellulose content is normally distributed.

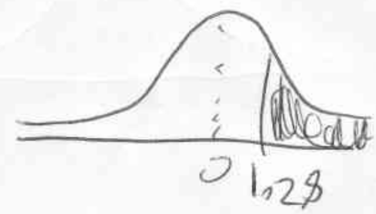
Problem 6.101

(a)



This area is the P-value.

$$Z = \frac{6.9 - 0}{55/\sqrt{104}} = 1.28$$



$$P \text{ value} = P(Z > 1.28)$$

$$= 1 - P(Z < 1.28)$$

$$= 1 - .8997$$

$$= .1003$$

(C) because the p-value is larger than $\alpha = .05$ we conclude that the null hypothesis can not be rejected. ie we have no evidence to show that the mean CEO compensation increased.

Again read Section 6.3

Key point of 6.3 is that tests aren't magical. They depend on assumptions (which may or not be true) and if you are not careful can be manipulated.