Lecture 21

A problem solving session on material from chapter 5

Some recommended problems

MWF 11–12		MWF 2–3	
5.55, 5.58 5.39, 5.61	discussed in class	5.15, 5.11 5.38, 5.40	discussed in class

Schedule

15 minutes attempting 1st question
10 minutes discussing solution.
15 minutes attempting 2nd question
10 minutes discussing solution

Work should be done in groups

Solutions For MWF 11–12

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Problem 5.55

"The number of high school dropouts in 25,000"
$\equiv X$
is a binomial random variable with $n = 25000$
$p = .121$

(a) For a binomial random variable

mean $\mu_X = np$

std dev $\sigma_X = \sqrt{np(1-p)}$

so mean $\mu_X = (25000)(.121) = 3025$

and standard deviation $\sigma_X = \sqrt{25000(.121)(1-.121)}$
$= 51.5653$ (4dp)

(b) The probability statement is

$P(X \geq 3500)$

Because the sample size is large

X is approximately Normal with mean $\mu = 3025$
and standard deviation $\sigma = 51.5653$

So

$$P(X \geq 3500) \approx P\left(\overbrace{\frac{X-3025}{51.5653} \geq \frac{3500-3025}{51.5653}}^{\text{This is the standardization step}}\right)$$

this is because of the normal approximation

$$= P(Z \geq 9.21)$$

$$= 1 - P(Z < 9.21)$$

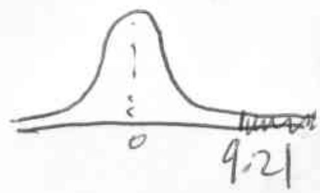
This symbol means "much much less"

From the table we know that

$$P(Z < 3.49) \ll P(Z < 9.21)$$

$$\Rightarrow .9998 \ll P(Z < 9.21)$$

$$\Rightarrow 1 - P(Z < 9.21) \ll .0002$$



Problem 5.58

X ≡ "The number of red blossoms in n plants"

is binomial random variable with $n = n$ $p = .75$

(a) $n = 8$

$$P(X=6) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

$$= \binom{8}{6}(.75)^6(1-.75)^{8-2}$$

$$= \frac{8!}{6!\,2!}(.75)^6(.25)^2 = .3115 \text{ (4dp)}$$

(b) $n = 80$

because X is a binomial r.v

$\mu_X = np = (80)(.75) = 60$

(c) Probability statement is

$P(X \geq 50)$

Because n is large and more exactly

$np = (80)(.75) = 60 > 10$

and

$n(1-p) = (80)(.25) = 20 > 10$

X is approximately normal with mean 60

and standard deviation $\sqrt{80(.75)(1-.75)} = 3.8730$

so

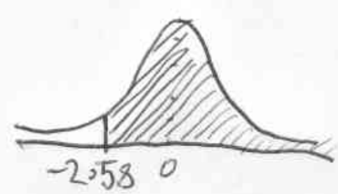
$$P(X \geq 50) \approx \overbrace{P\left(\frac{X-60}{3.8730} \geq \frac{50-60}{3.8730}\right)}^{\text{Standardization step}}$$

This is because of the approximation

$= P(Z \geq -2.58)$

$= 1 - P(Z < -2.58)$

$= 1 - .0049 = .9951$



Problem 5.39

Central limit theorem tells us that as sample size n increases the sampling distribution of $\bar{x}$ approaches normal with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$

since $\mu = 1.6$, $\sigma = 1.2$ $n = 200$

$\bar{x}$ has approximately normal distribution with mean 1.6, standard deviation $\frac{1.2}{\sqrt{200}} = 0.0849$

Probability statement

$$P(\bar{x} > 2) \approx P\Big(\underbrace{\frac{\bar{x} - 1.6}{.0849} > \frac{2 - 1.6}{.0849}}_{\text{standardization}}\Big)$$

normal approximation

$$= P(Z > 4.71)$$

note this is outside the limits of table.

$$= 1 - P(Z < 4.71)$$

$$\ll 1 - P(Z < 3.49)$$ largest value in table

$$\ll .0002$$

Problem 5.61

(a) Not binomial because n changes between vehicles (ie not every car has same size and same potential number of seats)

(b) Can't be normal because can't have half a person (nor negative people)

(c) Central limit theorem tells us that as n increases the distribution of $\bar{x}$ approaches normal with mean 1.5 and standard deviation $\frac{.75}{\sqrt{700}} = .0283$

(d) Need mean and std dev of $700\bar{x}$

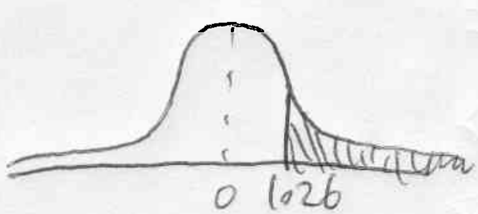
$$\mu_{700\bar{x}} = 700\mu_{\bar{x}} = 700(1.5) = 1050$$

$$\sigma_{700\bar{x}} = |700|\sigma_{\bar{x}} = 700\frac{(.75)}{\sqrt{700}} = 19.8431 \text{ (4dp)}$$

Probability is

approximation due to CLT

$$P(700\bar{x} > 1075) \approx P\left(\frac{700\bar{x} - 1050}{19.8431} > \frac{1075 - 1050}{19.8431}\right)$$



$$= P(Z > 1.26)$$

$$= 1 - P(Z < 1.26)$$

$$= 1 - .8962 = .1038$$

Solutions for MWF 2-3

Problem 5.15

(a) When sample size is large sampling distribution of $\hat{p}$ is approximately normal with mean $\mu_{\hat{p}} = p = .51$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.51)(.49)}{1005}}$

$= .0158$ (4dp)

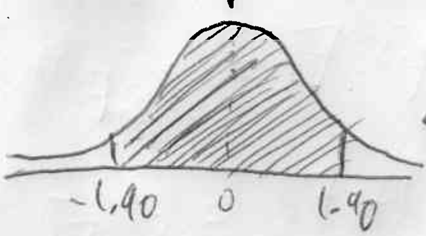
want

this is because of the approximation

$$P(.48 \leq \hat{p} \leq .54) \approx P\left(\frac{.48-.51}{.0158} \leq \frac{\hat{p}-.51}{.0158} \leq \frac{.54-.51}{.0158}\right)$$

the standardization step

$$= P(-1.90 \leq Z \leq 1.90)$$



$$= P(Z \leq 1.90) - P(Z \leq -1.90)$$

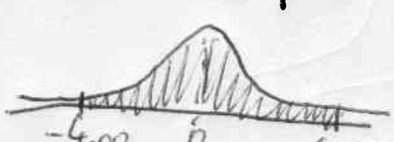
$$= .9713 - .0287$$

$$= .9426$$

(b) $p = .06$ $\quad \mu_{\hat{p}} = .06$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{.06(.94)}{1005}} = .0075$ notice this is smaller

want $P(.03 \leq \hat{p} \leq .09) \approx P\left(\frac{.03-.06}{.0075} \leq \frac{\hat{p}-.06}{.0075} \leq \frac{.09-.06}{.0075}\right)$

$$= P(-4.00 < Z < 4.00)$$



≈ 1 (ie probability increased)

Problem 5.11

For McGwire $X \equiv$ "number of HR in 509 at bats"

X is binomial (approximately) with $n = 509$ $p = .116$

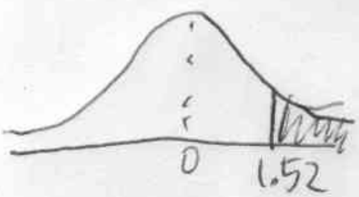
(a) $\mu_X = np = (509)(.116) = 59.044$

(b) $P(X \geq 70)$ ← probability statement

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{509(.116)(1-.116)}$$
$$= 7.22$$

because n is large X is approximately normal with mean and sd above)

$$P(X \geq 70) \approx P\left(\frac{X - 59.044}{7.22} \geq \frac{70 - 59.044}{7.22}\right)$$



$$= P(Z \geq 1.52)$$
$$= 1 - P(Z < 1.52)$$
$$= 1 - .9357$$
$$= .0643$$

(c) For Bonds $X \equiv$ "number of HR in 476 at bats"

X is binomial distributed with $n = 476$ $p = .0865$

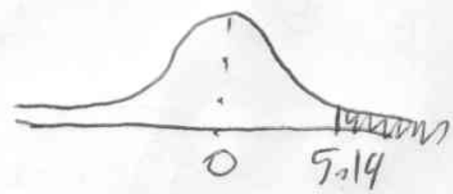
$$\mu_X = np = (476)(.0865) = 41.174$$

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{476(.0865)(1-.0865)}$$
$$= 6.1329 \text{ (4dp)}$$

X is approximately normal with mean 41.174 sd 6.1329

$$P(X \geq 73) \approx P\left(\frac{X - 41.174}{6.1326} \geq \frac{73 - 41.174}{6.1326}\right)$$

$$= P(Z \geq 5.19)$$



$$= 1 - P(Z < 5.19)$$

$$\ll 1 - P(Z < 3.49)$$

$$= .0002 \quad \text{(ie not very likely)}$$

Problem 5.38

(a) We know from the problem that X is exactly normally distributed with mean 55000 and standard deviation 4500. therefore the distribution of $\bar{X}$ is exactly normal with mean 55000 and standard deviation $\frac{4500}{\sqrt{8}} = 1590.99$

(b) $$P(\bar{X} \le 51800) = \underbrace{P\left(\frac{\bar{X}-55000}{1590.99} \le \frac{51800-55000}{1590.99}\right)}_{\text{standardization step}}$$

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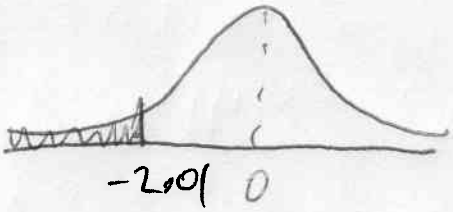
$$= P(Z \le -2.01)$$

$$= .0222$$



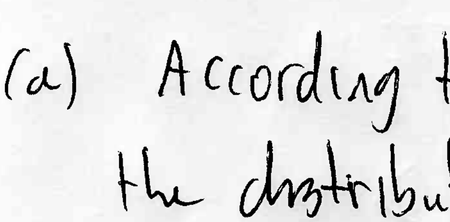
Problem 5.40

(a) According to the CLT as n increases the distribution of $\bar{x}$ approaches normal with mean 2.2 and standard deviation $\frac{1.4}{\sqrt{52}} = .1941$ (4dp)

(b) $$P(\bar{X} < 2) \overset{\text{CLT approximation}}{\approx} P\left(\frac{\bar{X}-2.2}{.1941} < \frac{2-2.2}{.1941}\right)$$

$$= P(Z < -1.03)$$

$$= .1515$$

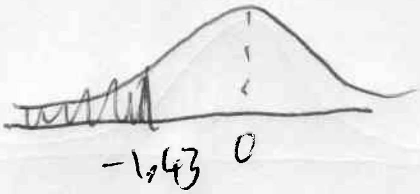


(c) Need mean and sd of $52\bar{X}$

$\mu_{52\bar{X}} = 52(2.2) = 114.4$ $\quad \sigma_{52\bar{X}} = (52)\frac{1.4}{\sqrt{52}} = 10.0955$

$$P(52\bar{x} < 100) \approx P\left(\frac{52\bar{x} - 114.4}{10.0955} < \frac{100 - 114.4}{10.0955}\right)$$

$$= P(z < -1.43)$$



$$= .0764$$