Lecture 14

More on the Normal distribution

Steps for finding probabilities

Step 1. Define the random variable and the conditions you are looking for mathematically

Step 2. Transform using $z = \frac{X - \mu}{\sigma}$ if necessary

Step 3. Sketch the standard normal distribution and the area you are looking for.

Step 4. Look up probabilities in the table

Examples

1. Cholesterol levels in women aged 20 to 34 are Normal with mean 185 mg/dL and standard deviation 39 mg/dL. A level above 240 mg/dL requires medical attention. What is the probability a randomly chosen women aged 20 to 34 needs medical attention for cholesterol?

Step 1 Let X = "cholesterol Level of women"

X is distributed normal with mean 185 sd 39.

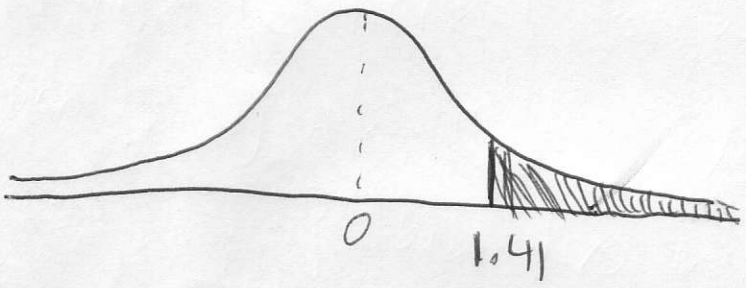
"need medical attention" $\equiv X > 240$

So want $P(X > 240)$

Step 2 $P(X > 240) = P\left(\frac{X - 185}{39} > \frac{240 - 185}{39}\right)$

$= P(Z > 1.41)$

Step 3



Step 4 $P(Z > 1.41) = 1 - P(Z < 1.41)$

$= 1 - .9207$

$= .0793$

2. Continuing the example above. Suppose that levels above 200mg/dL are considered borderline high. What is the probability a womens cholesterol

Level is between 200 mg/dL and 240 mg/dL

Step 1 $X =$ "cholesterol level of women"

X is distributed normal mean 185, sd 39

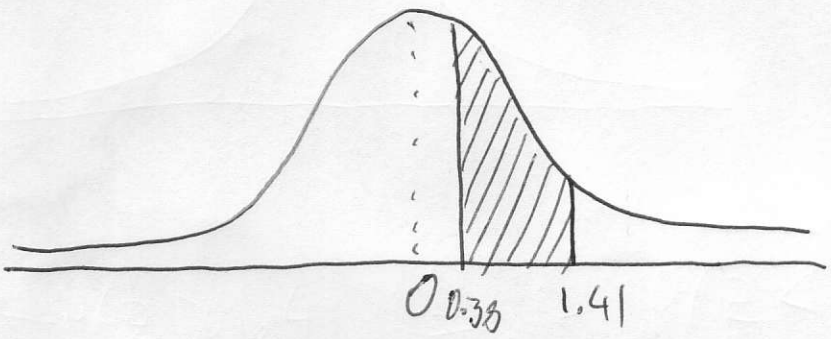
"borderline high" $\equiv 200 < X < 240$

so want $P(200 < X < 240)$

Step 2 $$P(200 < X < 240) = P\left(\frac{200-185}{39} < \frac{X-185}{39} < \frac{240-185}{39}\right)$$

$$= P(0.38 < Z < 1.41)$$

Step 3



Step 4 $$P(0.38 < Z < 1.41) = P(Z < 1.41) - P(Z < 0.38)$$

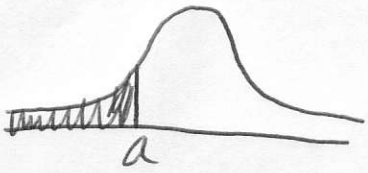
$$= .9207 - .6480$$

$$= .2727$$

General Cases

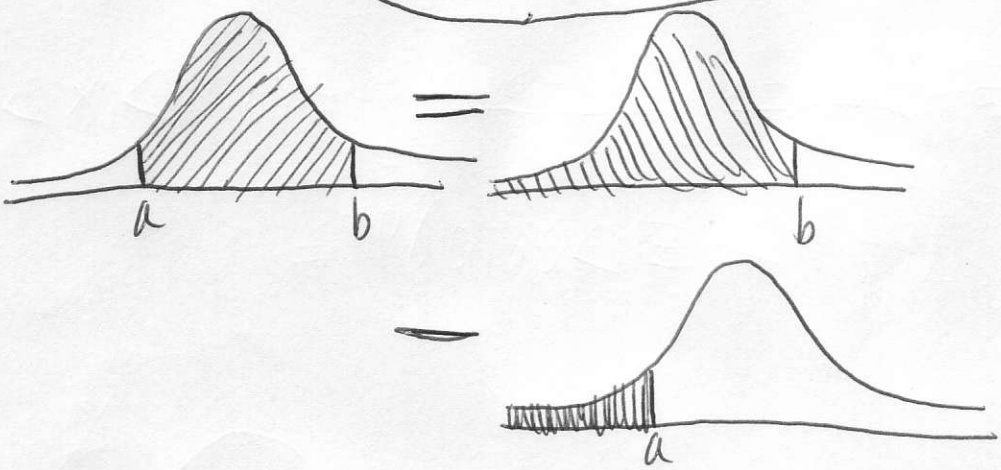
Let a, b be constants $a < b$. Let Z be a standard normal r.v.

$P(Z < a) = \ldots$ straight from table



$P(a < Z < b) = P(Z < b) - P(Z < a)$ get from table

ie



$P(Z > b) = 1 - P(Z < b)$ from table

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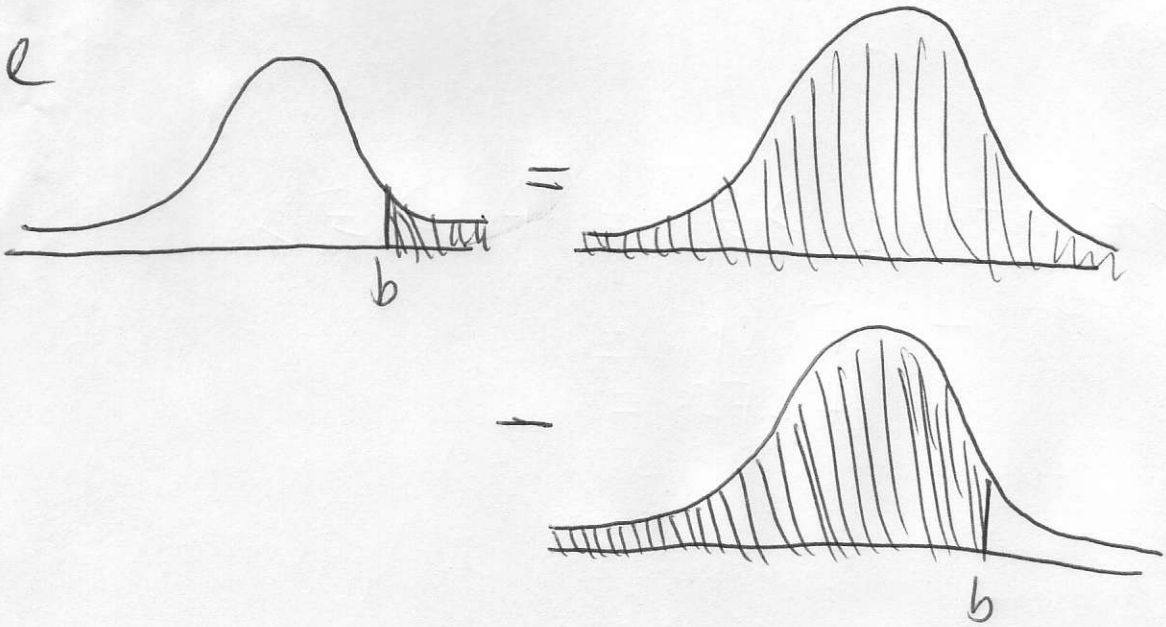
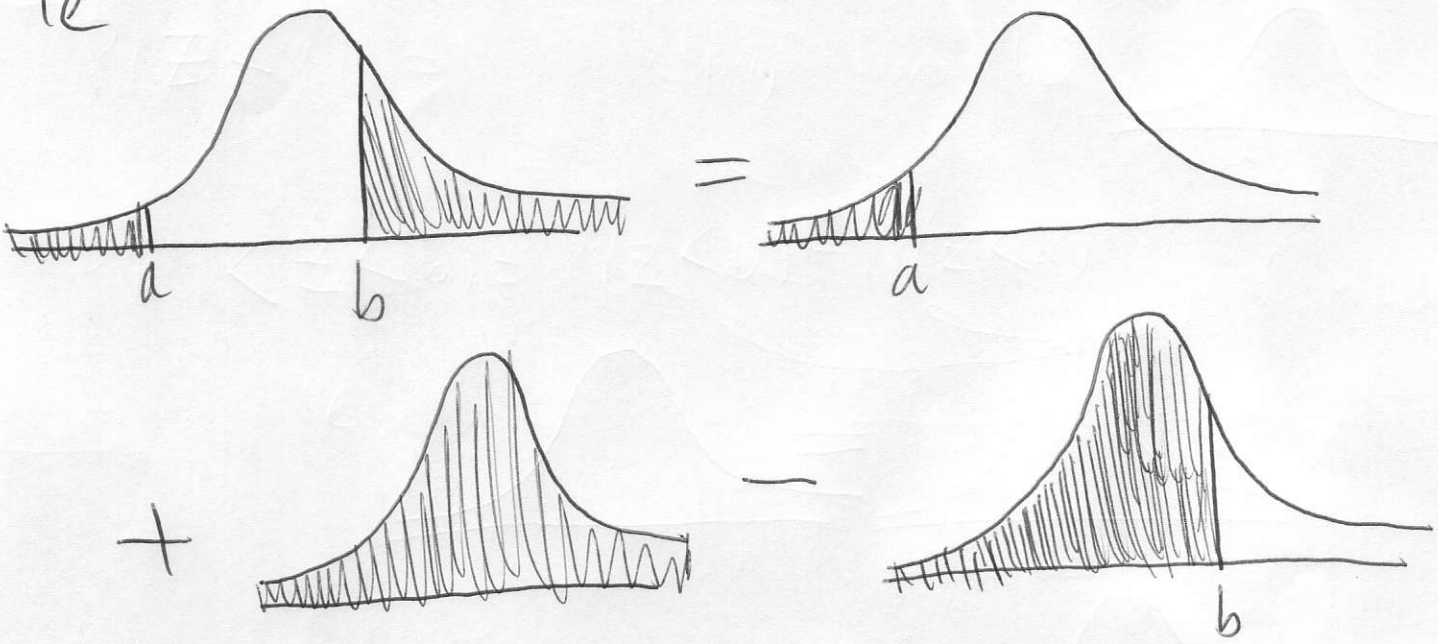


$P(\cancel{X} < a \text{ or } \cancel{X} > b) = P(\cancel{X} < a) + 1 - P(\cancel{X} < b)$

From table

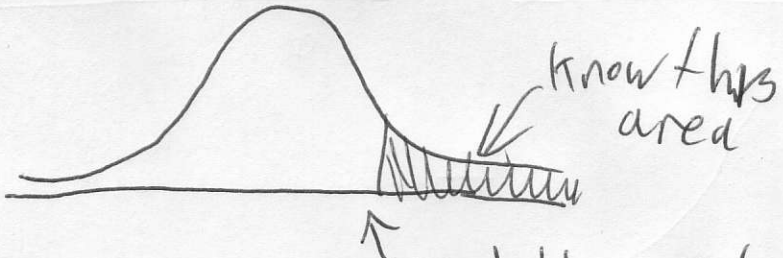
ie



Finding percentiles

This is somewhat the reverse of the previous situation. Know the area under the curve want to find the x value

eg



Steps to finding a percentile

Step 1 Define the random variable and the conditions you are looking for mathematically

Step 2 Sketch the normal distribution shading the area that you know

Step 3 Look up z in the table by locating the closest probability

Step 4 Unstandardize using

$$x = z\sigma + \mu$$

Example

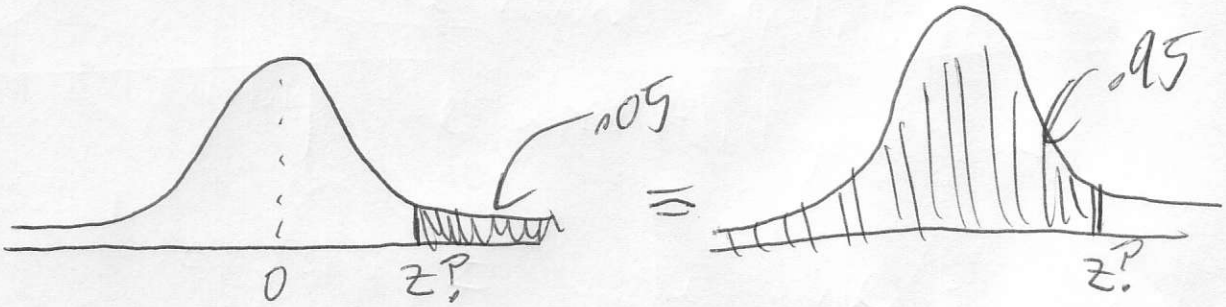
1. Continuing the example above. Above what cholesterol level would we find 5% of women aged 20 to 34

Step 1 $X =$ "cholesterol level of women"

X is normal mean 185 sd 34

want x for which $P(X > x) = .05$

Step 2



Step 3 Look for probability .95 in table

Closest values are 1.64 and 1.65. take the average so $z = 1.645$

Step 4 $x = (1.645)39 + 185$

$= 249.15$

2. Continuing example. Between what values do the central 95% of women lie?

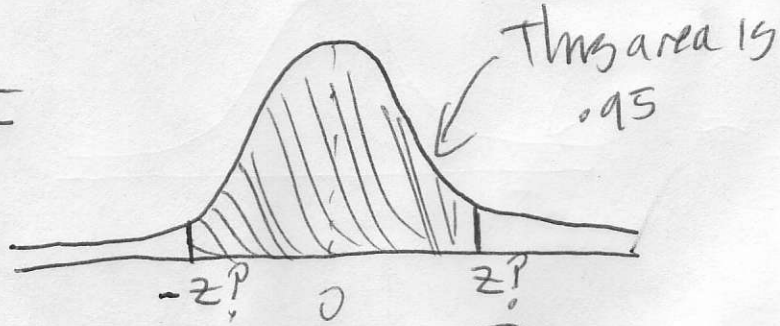
Step 1 X = "cholesterol level of women"

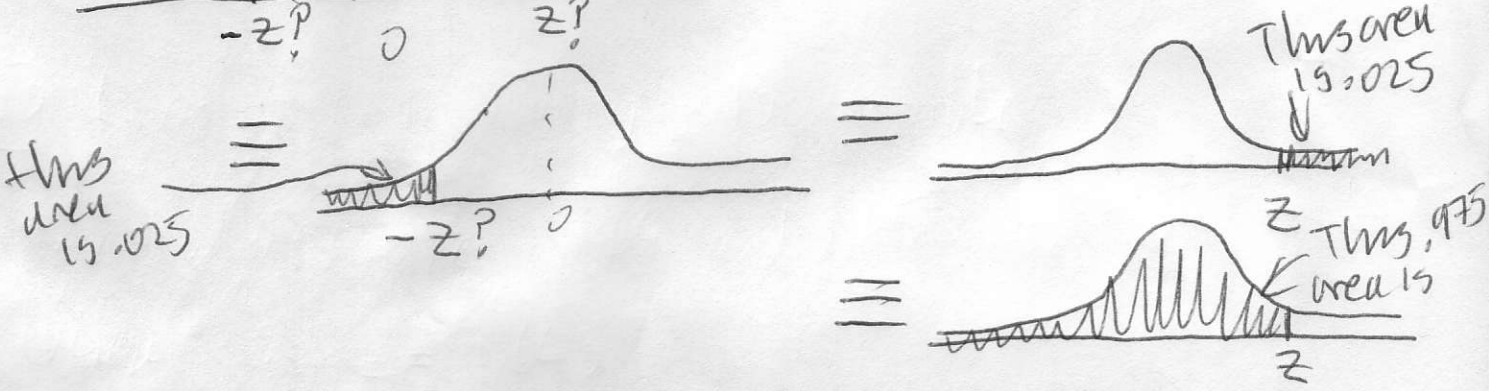
X is normal mean 185, sd 139

want $P(x_1 < X < x_2) = .95 \equiv P(-z < Z < z) = .95$

why $z, -z$? Symmetry

Step 2





Step 3 look for probability .975 (or .025) in table. (closest is $z = 1.96$ $(-z = -1.96)$

Step 4

$$x_1 = (1.96)39 + 185 = 261.4$$

$$x_2 = (-1.96)39 + 185 = 108.6$$