Lecture 13

1

Normal Distribution

Useful in many situations in many case the data is approximately normal. eg heights of adults about the same age, test scores, ...

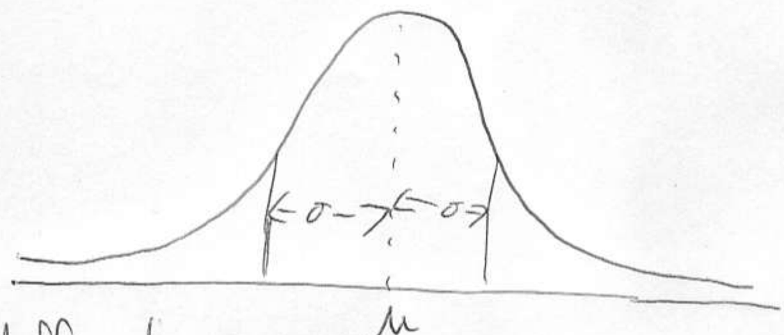
Continuous distribution. Two parameters

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

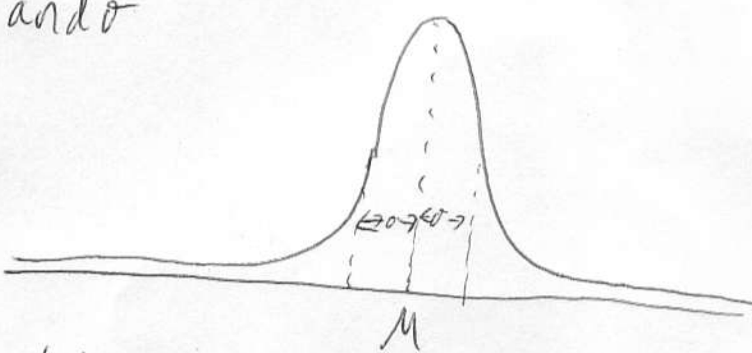
function for density curve

μ – mean

σ – standard deviation



different μ and σ

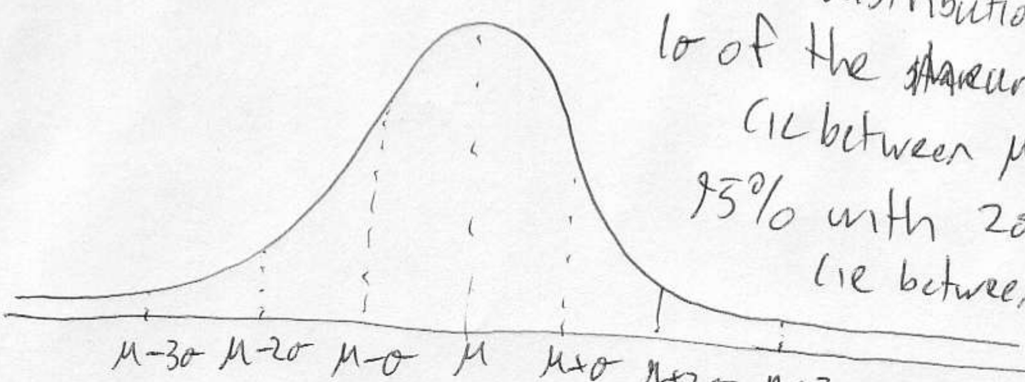


μ controls the center

σ controls the spread

Note that approximately 68% of observations in a normal distribution fall within 1σ of the ~~m~~ mean (ie between $\mu-\sigma$ and $\mu+\sigma$)

95% with 2σ of the mean (ie between $\mu-2\sigma$ and $\mu+2\sigma$)

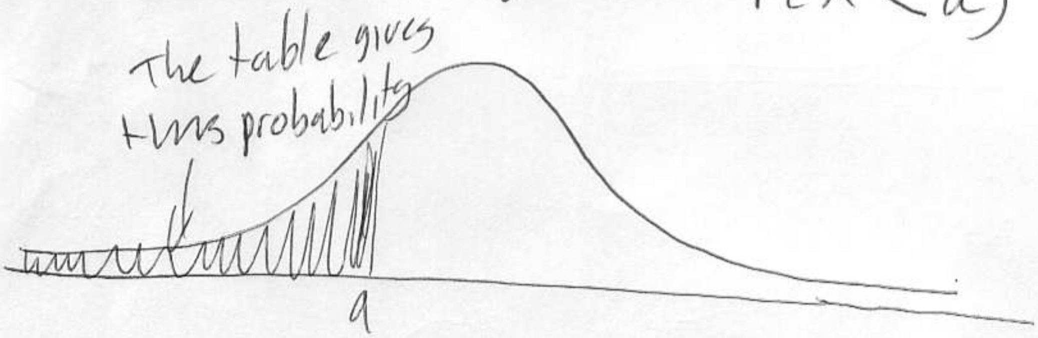


99.7% within 3σ of the mean (ie between $\mu-3\sigma$ and $\mu+3\sigma$)

A special case

The Normal distribution with mean 0 and std dev 1 is known as the standard Normal Distribution. As discussed last time areas under density curves give probabilities. There is a special table which gives these probabilities for the standard normal distribution (see attached table or in textbook).

Suppose X is a standard normal random variable and "a" is a constant number (eg 1.5, −2.34, ...) then the table gives $P(X < a)$



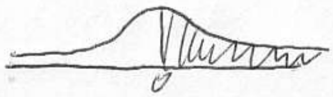
eg $P(X < -2.23) = .0129$ (Find this in the table)

Some special probabilities and relationships

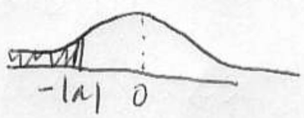
X is standard normal random variable

$P(X<0)=0.5$

$P(X>0)=0.5$



$P(X<-|a|)=P(X>|a|)$ where "a" is a constant



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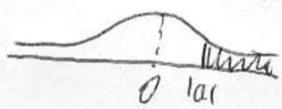


(this is from symmetry of normal distribution)

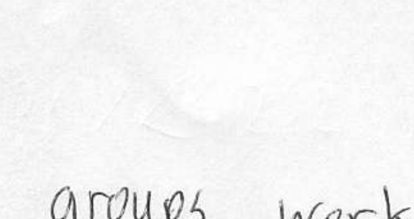
$P(X<\infty)=1$

$P(X>-\infty)=1$

$P(X>\infty)=0$

$P(X<-\infty)=0$

$P(X>a)=1-P(X<a)$ where "a" is a constant



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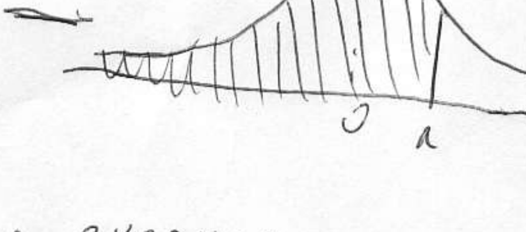

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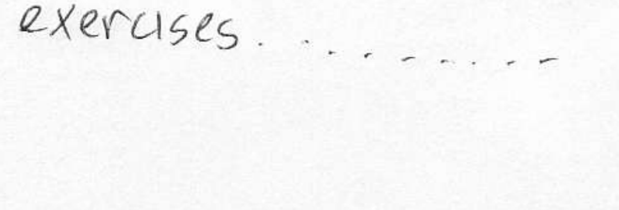

In groups work on exercises. . . . . . .

Standardizing

If you have data from normal distribution with mean μ and standard deviation σ you can "standardize" it using the equation

$$Z = \frac{X - \mu}{\sigma}$$

This allows you to use the standard normal table to find probabilities for any normal distribution

eg Suppose X is Normally distributed with mean 25 and stdev 5. what is $P(X > 30)$?

$$P(X > 30) = P\left(\frac{X - 25}{5} > \frac{30 - 25}{5}\right)$$

$$= P(Z > 1.0)$$

$$= 1 - P(Z < 1.0)$$

$$= 1 - .8413 = 0.1587$$

Now work on the second set of problems. Again in groups.