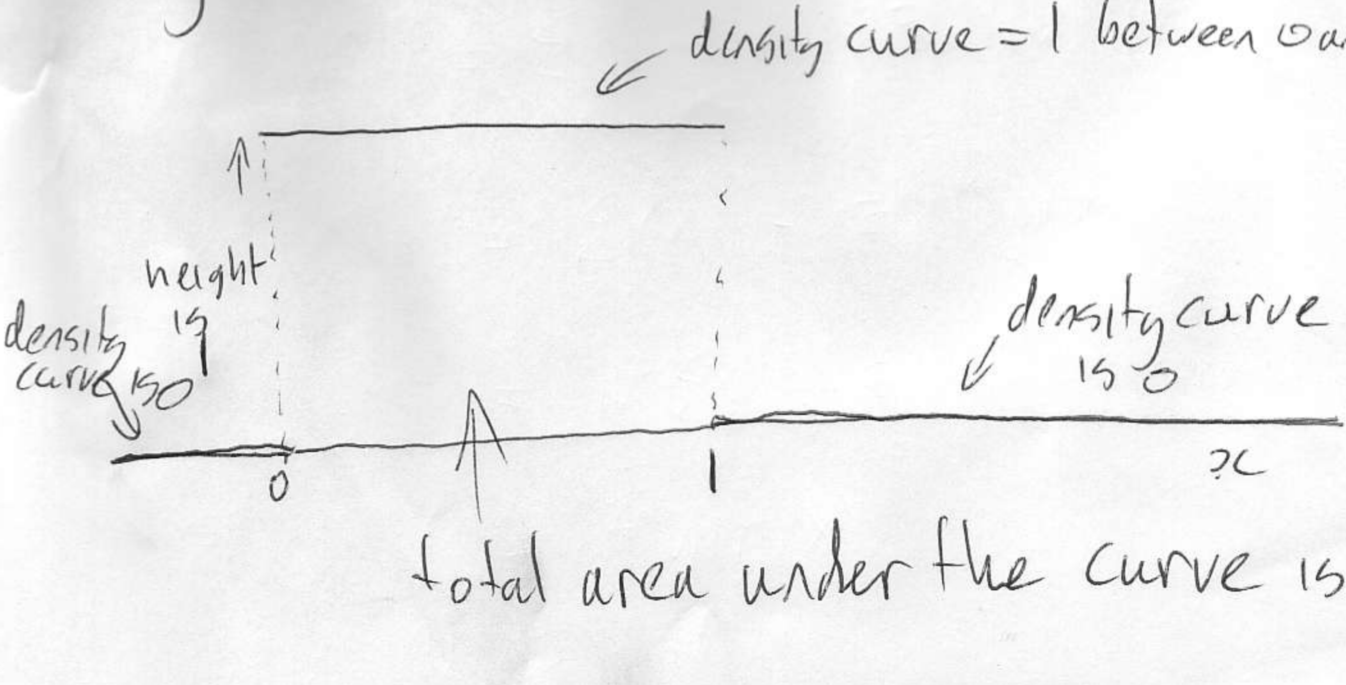
Lecture 12

As mentioned last lecture continuous random variables can take on any value in a range of numbers. The probability distribution of a continuous r.v is described by a <u>density curve</u>. The probability of an event is the area under the curve above the values of the random variable that make up the event.

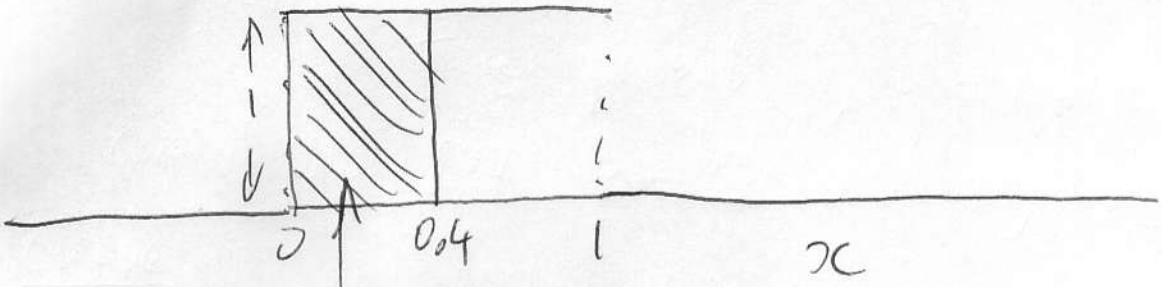
Density curves have exactly area 1 under the curve. ie total probability of 1

eg Uniform distribution between 0 and 1



Suppose r.v. X has uniform distribution.

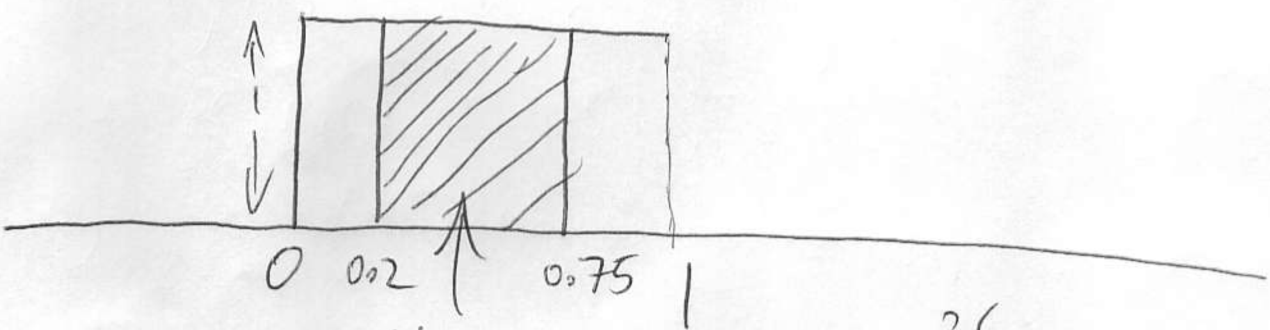
What is $P(X \leq 0.4)$?



this area is the probability

area $= 0.4 \times 1 = 0.4 = P(X \leq 0.4)$

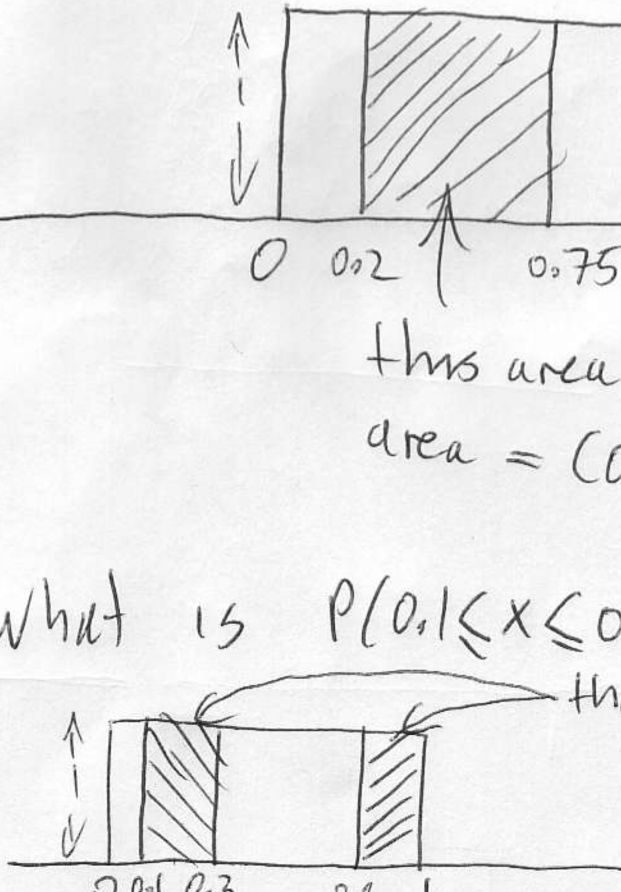
What is $P(0.2 \leq X \leq 0.75)$



this area is the probability

area $= (0.75 - 0.2) \times 1 = 0.55 = P(0.2 \leq X \leq 0.75)$

What is $P(0.1 \leq X \leq 0.3 \text{ or } X \geq 0.8)$?



this area is the probability

area $= (0.3 - 0.1)(1) + (1 - 0.8)(1)$

$= 0.2 + 0.2 = 0.4$

<u>Note</u> Continuous probability distributions assign probability 0 to any individual outcome so for example $P(X = 0.4) = 0$ in the previous example.

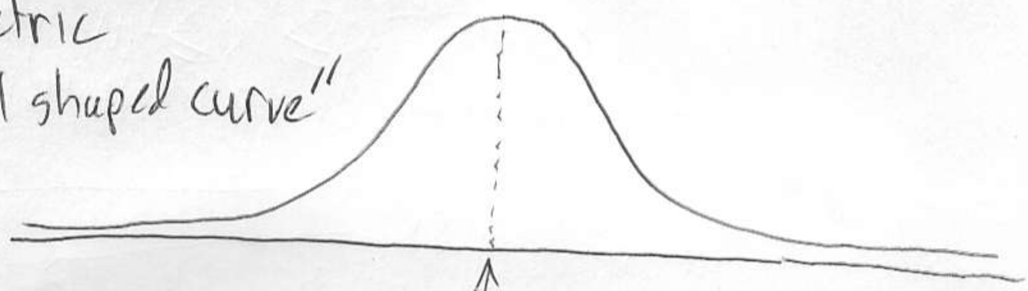
Think of this as being true because there is no area under the curve at a single data point.

<u>A Special Continuous Distribution</u>

The <u>normal distribution</u>

two parameters mean μ
std dev σ

symmetric
"bell shaped curve"



mean value μ

We will take more time to learn about the normal distribution next lecture. It will be very important that you learn how to compute normal distribution probabilities. You will need to use the normal distribution many times between now and the end of this class.

The mean of a discrete random variable

Value of X	x_1	x_2	x_3	.....	x_K
Probability	p_1	p_2	p_3	.....	p_K

probability distribution

The mean of r.v X is given by

$$\mu_X = x_1 p_1 + x_2 p_2 + \cdots\cdots + x_K p_K$$

$$= \sum_{i=1}^{K} x_i p_i$$

eg roll 2 fair 4-sided dice X = "sum of two rolls"

Value of X	2	3	4	5	6	7	8
Probability	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$$\mu_X = 2\left(\frac{1}{16}\right) + 3\left(\frac{2}{16}\right) + 4\left(\frac{3}{16}\right) + 5\left(\frac{4}{16}\right) + 6\left(\frac{3}{16}\right) + 7\left(\frac{2}{16}\right) + 8\left(\frac{1}{16}\right)$$

$$= \frac{2 + 6 + 12 + 20 + 18 + 14 + 8}{16} = \frac{80}{16}$$

$$= 5$$

The variance of a discrete random

The variance of a discrete r.v. is

$$\sigma_X^2 = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_k - \mu_X)^2 p_k$$

$$= \sum_{i=1}^{k} (x_i - \mu_X)^2 p_i$$

eg Roll 2 fair 4-sided dice. X = "sum of two rolls"

$$\sigma_X^2 = (2-5)^2\left(\frac{1}{16}\right) + (3-5)^2\left(\frac{2}{16}\right) + (4-5)^2\left(\frac{3}{16}\right) + (5-5)^2\left(\frac{4}{16}\right)$$

$$+ (6-5)^2\left(\frac{3}{16}\right) + (7-5)^2\left(\frac{2}{16}\right) + (8-5)^2\left(\frac{1}{16}\right)$$

$$= \frac{9 + 8 + 3 + 0 + 3 + 8 + 9}{16} = 2.5$$

Rules about means

1. If X is a random variable and a and b are fixed numbers

$$\mu_{a+bX} = a + b\mu_X$$

2. If X and Y are r.v. then

$$\mu_{X+Y} = \mu_X + \mu_Y$$

eg Let $Y = 2X - 3$ X = "sum of two rolls of 4 sided dice"

as before

$$\mu_Y = \mu_{2X-3} = -3 + 2\mu_X$$
$$= -3 + 2(5)$$
$$= 7$$

Let Z = "sum of two rolls of a 6-sided dice"

It is easy to show that $\mu_Z = 7$ using the technique described earlier this lecture

- what is the mean of $X+Y$?

$$\mu_{X+Y} = \mu_X + \mu_Y = 5 + 7 = 12$$

Rules for Variances

1. If X is a r.v and a and b are constants then

$$\sigma^2_{a+bX} = b^2 \sigma_X^2$$

2. If X and Y are independent r.v then

$$\sigma^2_{X+Y} = \sigma_X^2 + \sigma_Y^2$$
$$\sigma^2_{X-Y} = \sigma_X^2 + \sigma_Y^2$$

eg $Y = 2X - 3$ let X = "sum of two rolls of a ~~six~~ four sided dice"

$$\sigma_Y^2 = \sigma_{2X-3}^2 = 2^2\sigma_X^2 = 4\sigma_X^2 = 4(2.5) = 10$$

Let Z be "sum of two rolls of a six sided dice"

$$\sigma_{Z+X}^2 = \sigma_Z^2 + \sigma_X^2$$

it can be shown that $\sigma_Z^2 = 5.8\dot{3}$

so $\sigma_{Z+X}^2 = 5.83 + 2.5 = 8.33$.

The book shows the formula for σ_{X+Y}^2 if X and Y are 2 r.v but not independent. You may read those formula, but will not need them for this class.