Lecture 11

A <u>random variable</u> is a variable whose value is a numerical outcome of a random phenomenom. Usually denoted using capital letters eg $X, Y, \ldots$ (lowercase $x, y, \ldots$ are possible values of the random variable).

eg toss two coins

$$S = \{HH, HT, TH, TT\}$$

Let
$\left.\begin{array}{l} \text{random} \\ \text{variables} \end{array}\right\{$
- X be number of heads
- Y be number of tails
- Z be number of heads minus number of tails
- V be 1 if two of same face, 0 if differ

Outcome	HH	HT	TH	TT
X	2	1	1	0
Y	0	1	1	2
Z	2	0	0	-2
V	1	0	0	1

In other words a random variable takes an outcome and turns it into a number.

Two main types of random variables

Discrete - only a finite number of possible values for random variable

Continuous - can take any value in an interval of numbers

eg Discrete - The number of girls in a randomly chosen family with 3 children

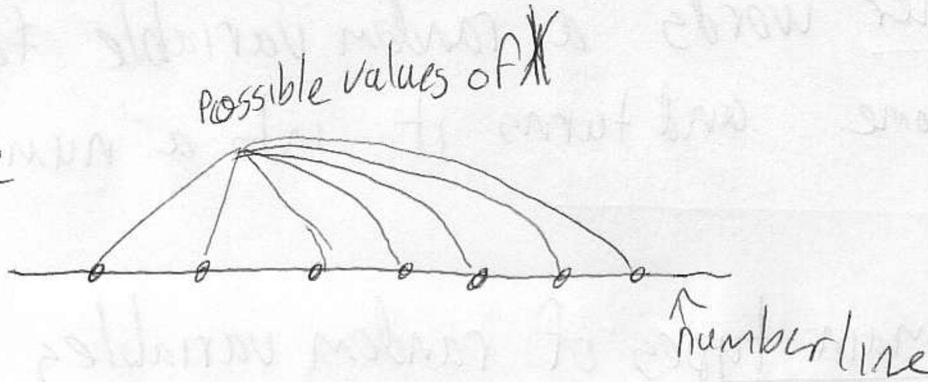
- The number of defective parts off an assembly line
- The sum of the numbers on two rolls of a dice

⋮

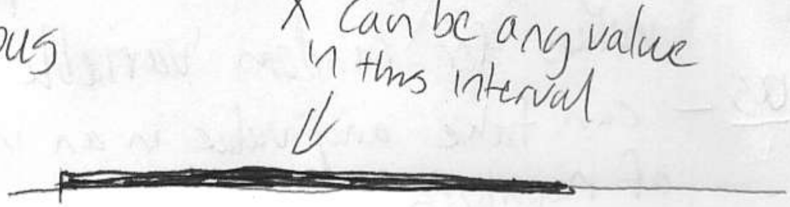
Continuous - The height of a randomly selected person

- The pH of a randomly selected soil sample
- the distance a long jumper leaps.

Discrete



Continuous



The probability distribution of a discrete random variable is a list of the possible values and their probabilities

eg

Value of X	Probability
x_1	p_1
x_2	p_2
x_3	p_3
$\vdots$	$\vdots$
x_K	p_K

The probabilities must satisfy two requirements

1. $0 \leq p_i \leq 1$ ie. probabilities are between 0 and 1
2. $\sum_{i=1}^{K} p_i = p_1 + p_2 + p_3 + \cdots + p_{K-1} + p_K = 1$

   ie the total probability equals 1

Note that the probability of any event is given by adding the probabilities p_i of the x_i making up the event

eg roll 2 fair 4 sided dice

let $X =$ sum of two rolls

	roll 2			
	(1,1)	(1,2)	(1,3)	(1,4)
	(2,1)	(2,2)	(2,3)	(2,4)
roll 1	(3,1)	(3,2)	(3,3)	(3,4)
	(4,1)	(4,2)	(4,3)	(4,4)

equally likely outcomes so the probability of each outcome is $\frac{1}{N} = \frac{1}{16}$

X	2	3	4	5	6	7	8
probability	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

what is the probability that X is 5 or bigger

$$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8)$$

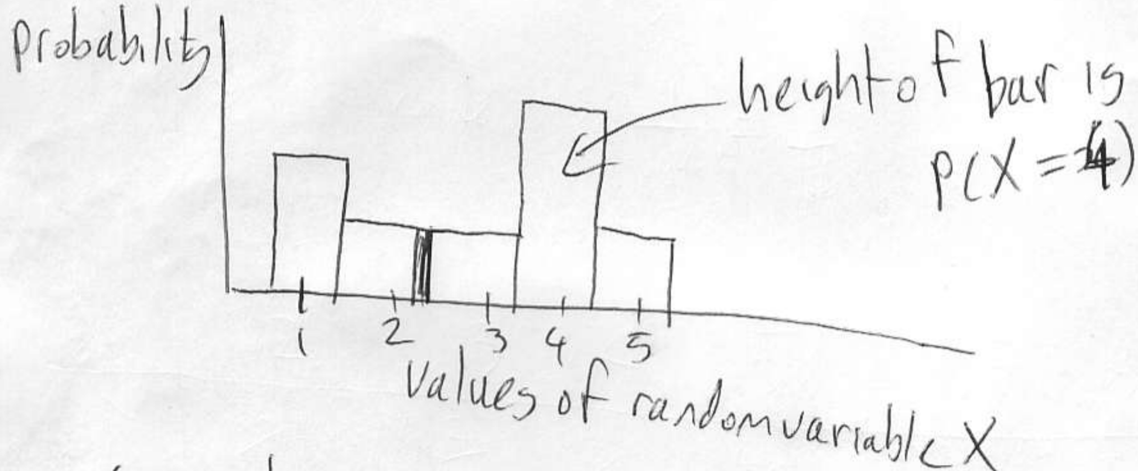
$$= \frac{4}{16} + \frac{3}{16} + \frac{2}{16} + \frac{1}{16} = \frac{5}{8}$$

what is the probability that X is prime

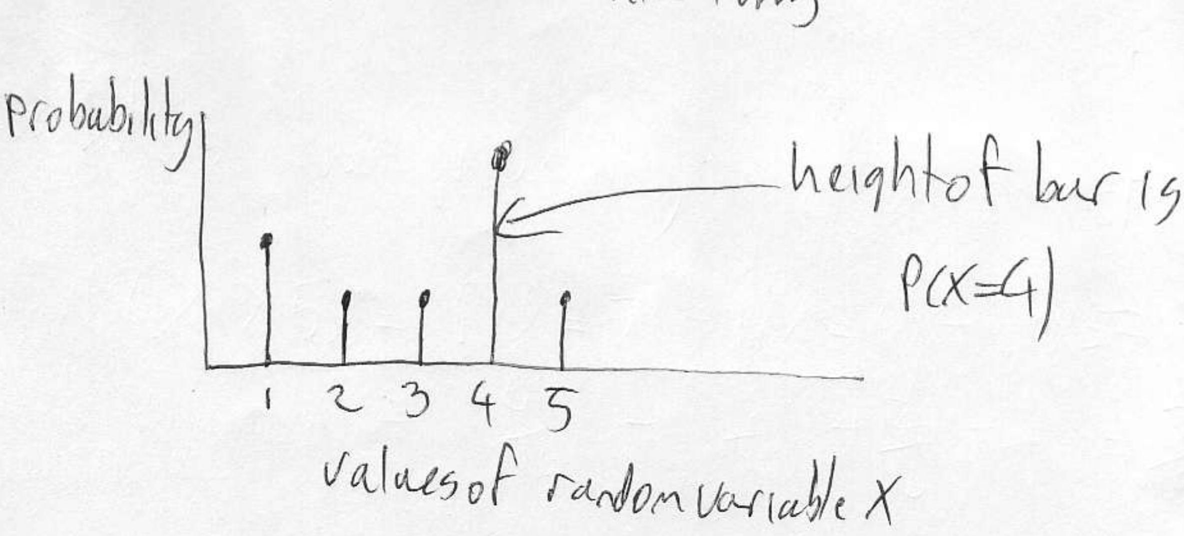
$$P(X \text{ is prime}) = P(X=2) + P(X=3) + P(X=5) + P(X=7)$$
$$= \frac{1}{16} + \frac{2}{16} + \frac{4}{16} + \frac{2}{16} = \frac{9}{16}$$

<u>Visual representation of a discrete probability distribution</u>

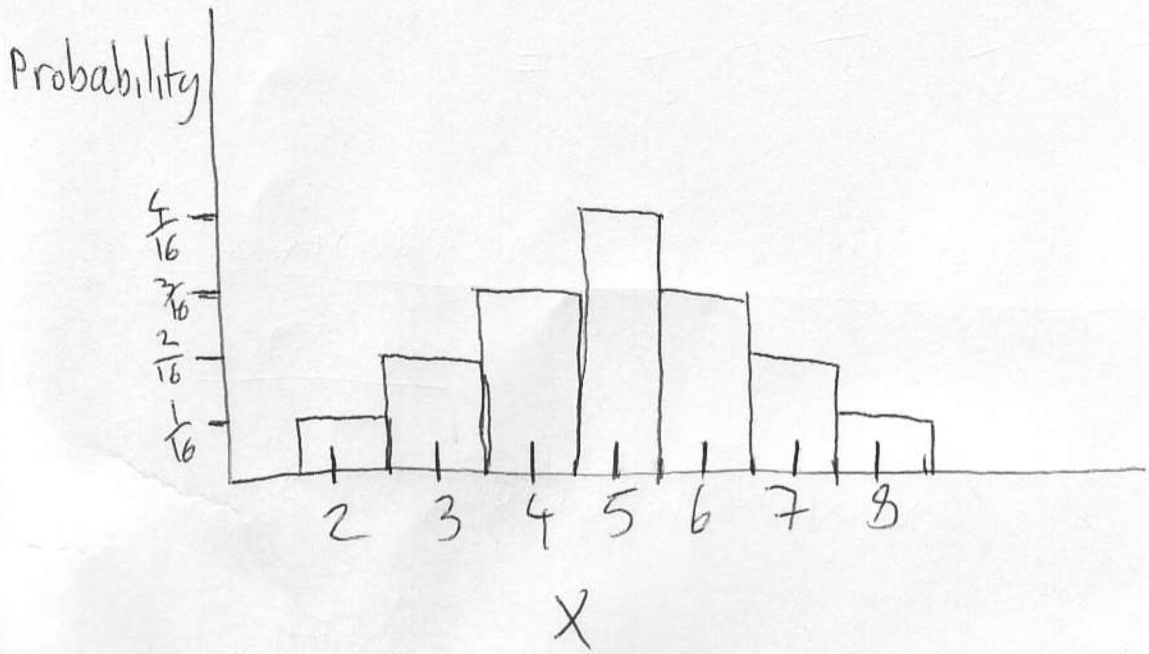
Use a probability histogram



Some times drawn like this



Returning to the 2 four side dice
with x = sum of two rolls



or