Continuing Independence .....

eg Roll two dice

A = "Get a 4, 5, 6 on first roll"

B = "Sum is 4 or 7"

$P(A) = \frac{18}{36} = \frac{1}{2}$ (see last time)

$P(B) = \frac{6}{36} + \frac{3}{36} = \frac{9}{36}$

$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$P(A)\,P(B) = \left(\frac{1}{2}\right)\left(\frac{9}{36}\right) = \frac{9}{72} = \frac{1}{8} \neq \frac{1}{2} = P(A \cap B)$.

So A and B are not independent. They are <u>dependent</u>.

Common Sampling situations

Sampling with replacement

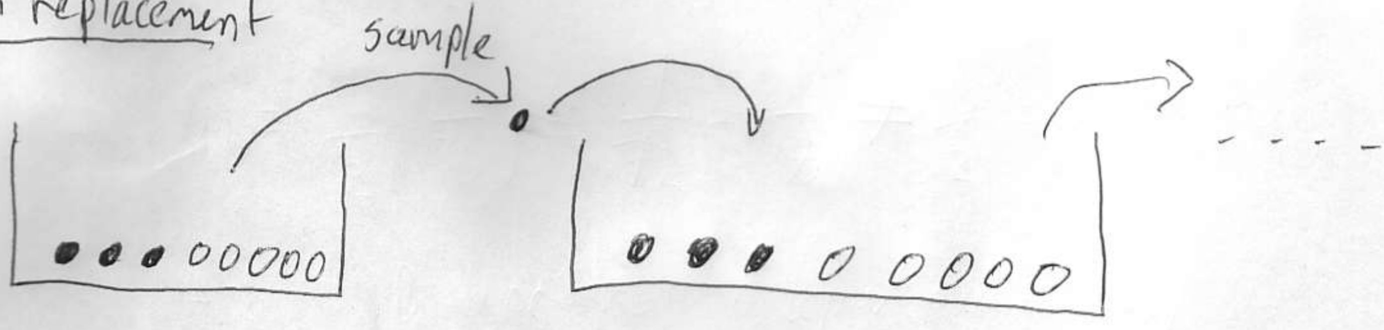
Once an item has been sampled it is placed back in the sampling space and may be sampled again. In this situation events are independent

sampling without replacement

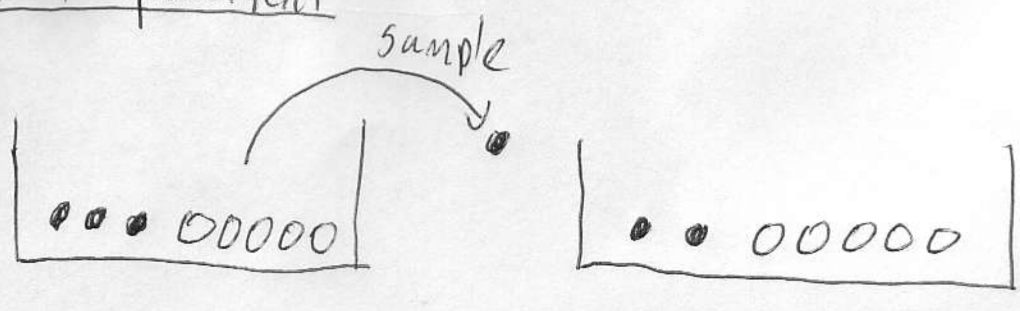
Once an item has been sampled it cannot be selected from the sampling space again. In this situation events are dependent.

In pictures:

with replacement



without replacement



Example

Consider a box contain 3 red balls and 4 white balls. what is the probability of getting 2 red balls if we sample two balls at random

(a) with replacement?
(b) without replacement?

(a) P(two red balls) = P(red ball on first draw and red ball on second draw)

$$= P(\text{red on } 1)\,P(\text{red on } 2) \quad (\text{independent})$$

$$= \frac{3}{7} \times \frac{3}{7}$$

$$= \frac{9}{49}$$

"conditional probability"

(b) P(two red balls) = P(red on 1 and red on 2)

$$= P(\text{red on } 1) \times P(\text{red on } 2 \text{ given red on } 1)$$

$$= \frac{3}{7} \times \frac{2}{6}$$

$$= \frac{6}{42} = \frac{1}{7}$$

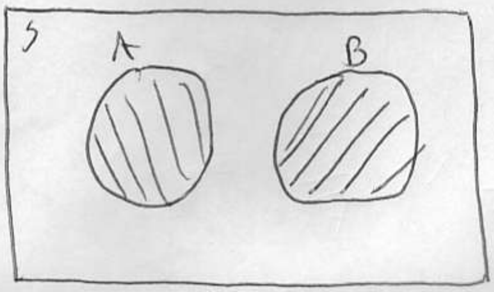
Some set operations

Union — the collection of ~~events~~ outcomes in at least one member of the events making up the collection. Symbol $\cup$. Means the same as "or"

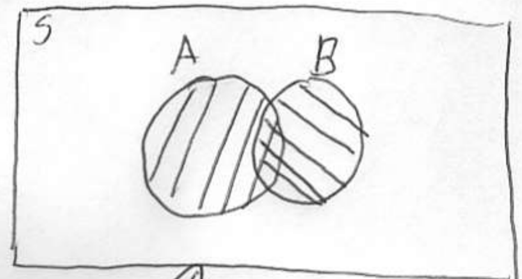
eg $A \cup B$ is the set of all outcomes in either event A alone, or B alone or both A and B.

Venn diagrams

A and B disjoint



A and B not disjoint

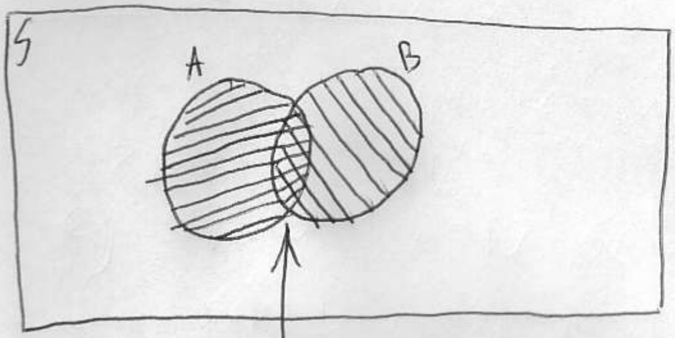


Anything shaded above is part of $A \cup B$

Intersection — the collection of outcomes that are simultaneously in all of the events making up the collection. Symbol $\cap$. Means the same as "and"

eg $A \cap B$ is the set of outcomes in both A and B.

Venn diagram



only the doubly shaded area is part of $A \cap B$

Recall that for disjoint events

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

For non disjoint events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

why? see the Venn diagram above. (the intersection is counted twice. Once as part of A and once as part of B so we need to subtract it off the total.

Conditional Probability

As we saw in the context of sampling without replacement sometimes if we know that one event has occured the probability of another event occuring changes. This is known as conditional probability. Notation: $P(A|B)$ read as the probability of A given that B occurred.

eg roll two dice

A = "Get a 4,5,6 on first roll"
B = "sum is 4 or 7"

Suppose we know that A occured. This means that the sum cannot possibly be 4, therefore the only outcomes that are possible are (6,1), (5,2), (4,3)

$$P(B|A) = \frac{3}{18} = \frac{1}{6} \quad \text{compare with } P(B) = \frac{9}{36}$$

#ways to get 4,5,6 on first roll!

Formula for conditional probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Check

$$\frac{P(A \cap B)}{P(A)} = \frac{1/12}{1/2} = \frac{2}{12} = \frac{1}{6} = P(B|A) \checkmark$$