Math 124 Lecture 5/6 Dr Bolstad
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Relationships between two variables

We have talked about how to inspect the distribution of a single variable, both graphically and using summary statistics. However, often we measure more than one variable for each individual eg height and weight, age and number of car accidents in the last 5 years, cholesterol and heart disease. If is useful to look at the relationship between variables in many contexts. We say that two variables, each measured on the same set of individuals, are *associated* if some values of one variable are more likely to be paired with some values of the other variable. eg higher values of variable 1 are more likely to be recorded with lower values of variable 2. Note that an association is only a tendency. It is not an ironclad rule (ie there may be perfectly sensible exceptions), and it does not prove that there is a causal relationship between the two variables that are associated.

We often describe variables as being either:

1. *Response variable*: a variable which measures the outcome of a study.
2. *Explanatory variable*: a variable that causes or explains the changes in the response variable

When we carryout an experiment where we control one varibale it is easy to say which is the explanatory variable (the one which we can set) and which is the response (the one that we measure). eg consider an experiment using a blood pressure medicine. The experimenter can control the level of the blood pressure medicine (the expalantory variable) and then measures the initial and final blood pressure levels of the patients after some period of time to get the change in blood presure (the response variable).

However, many times you only have observational data and therefore it is not clear as to what is the explantory variable and what is the response variable. In cases like this we generally choose based on what we are going to do with the data. For instance maybe we want to use one variable to predict values of the other variable. Sometimes we are helped by the temporal order in which things are measured (ie one variable happens before the other). For example suppose we measure students high school GPA and college GPA. While a high school GPA does not cause a college GPA, it may well be useful in helping predict it. Since high school is before college, we would treat the high school GPA as the explanatory variable and the college GPA as the response variable.

It is often the case that an unmeasured variable (we call this a "*lurking variable*") often explains or is the vause of both the observed variables. For example the hours studied per week could help explain both high school and college GPA.

Scatterplots

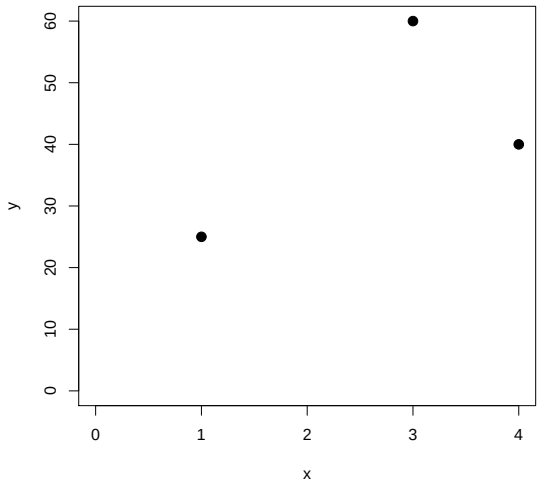
A scatterplot shows the relationship between two quantitative variables each measured on the same set of individuals eg measure both the height and weight for each person using a sample of 20 individuals. One variable falls on the horizontal axis (traditionally this is the explanatory variable) and one variable falls on the vertical axis (traditionally this is the response variable). Each individual is plotted as a point, with the location determined by the values of the two variables.

Example

Suppose we have 3 individuals and we have measured two variables x and y for each of these individuals. Our data table is the following

x	y
4	40
3	60
1	25

A scatterplot of this data looks like the following:



Interpreting a scatterplot

When examining a scatterplot there is several things you might what to consider, including:

- Overall Pattern

  Form

No clear pattern

Linear

Curve (non-linear)

Direction

Increasing

Decreasing

Strength

Strong

Weak

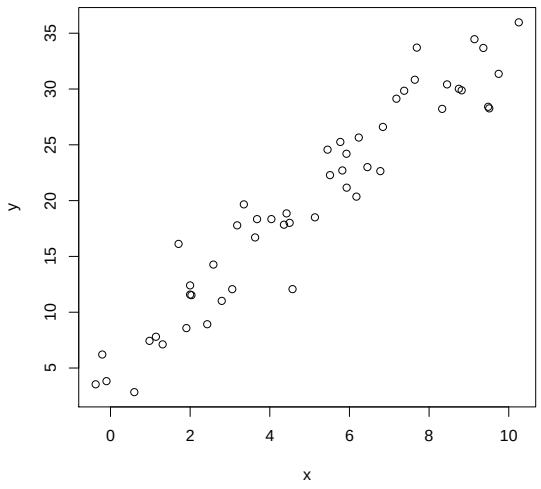
- Deviations from the pattern

  Outliers

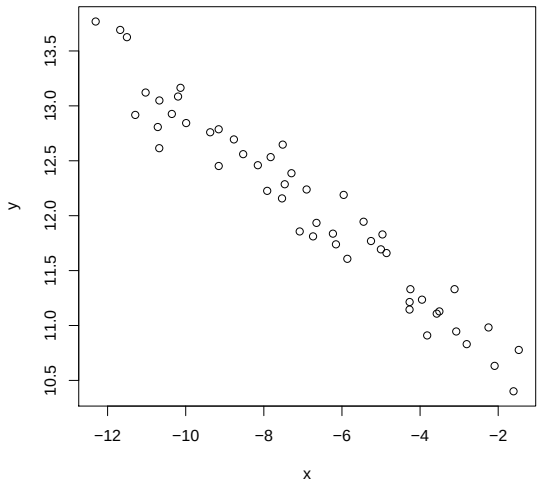
Direction

Direction is typically about deciding whether there is an increasing or decreasing relationship between the two variables being examined.

1. We say that the relationship between two variables is a *positive association* if we observe that as the value of one variable increases so does the value of the other variable. A typical scatterplot in this case looks something like the following:



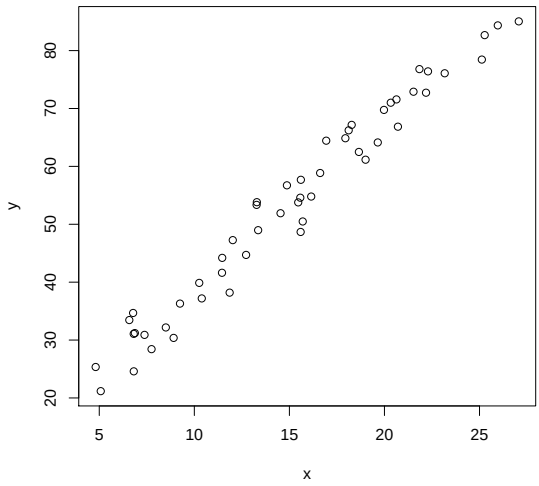
2. We say that the relationship between two variables is a *negative association* if we observe that as the value of one variable increases the value of the other variable decreases. A typical scatterplot in this case looks something like the following:



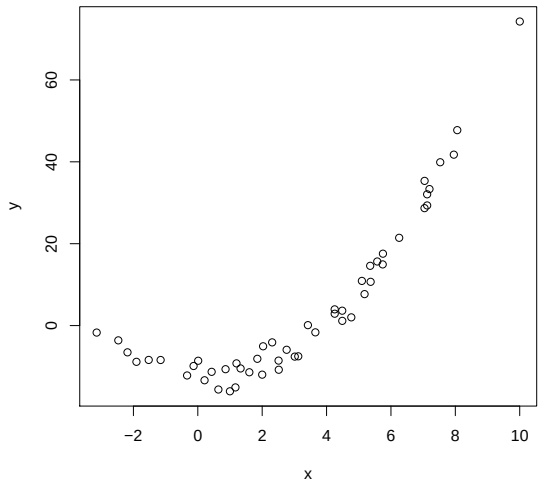
Form

Form is about determining the shape of the relationship between the two variables being examined.

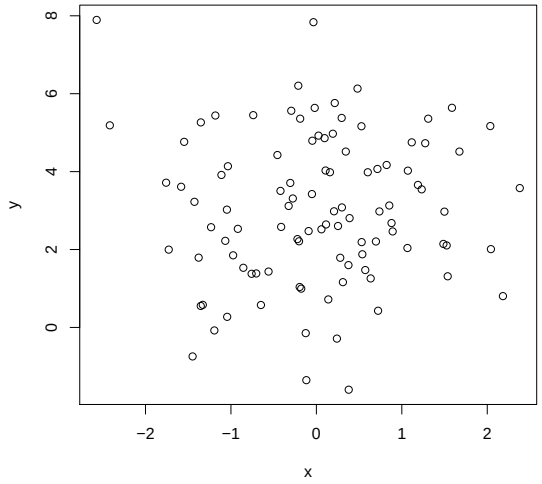
1. We say that the form is *linear* if the points seem to lie along a line



2. We say that the form is *non-linear* if the points seem to lie along a curve of some kind



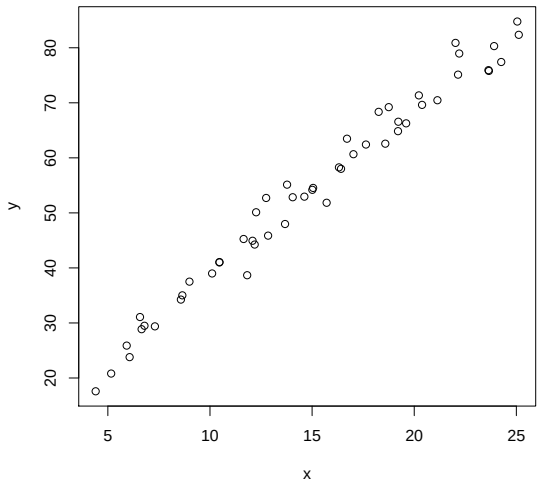
3. We say that the form has *no pattern* if there does not appear to be a clear relationship between the two variables.



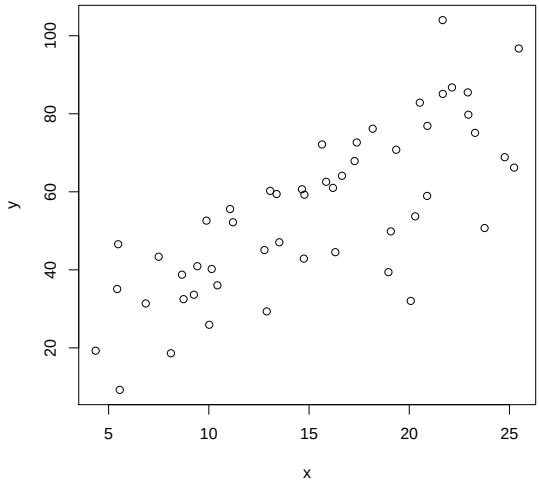
Strength

Strength is about how strongly the points on the scatterplot seem to follow the particular form

1. We say that the relationship is *strong* if the points seem to follow a specific form very closely.

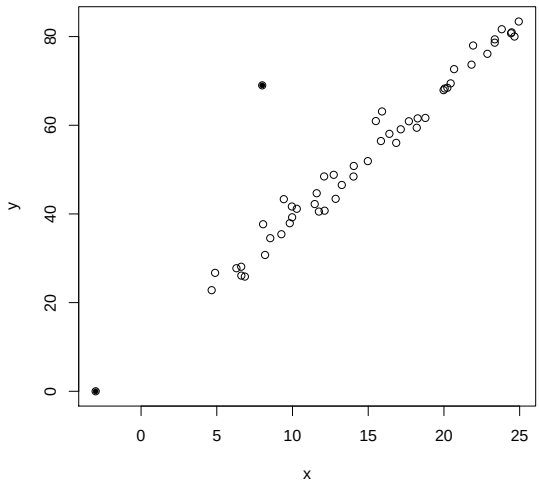


2. We say that the relationship is *weak* if the points seem to follow a specific form but not cloesly



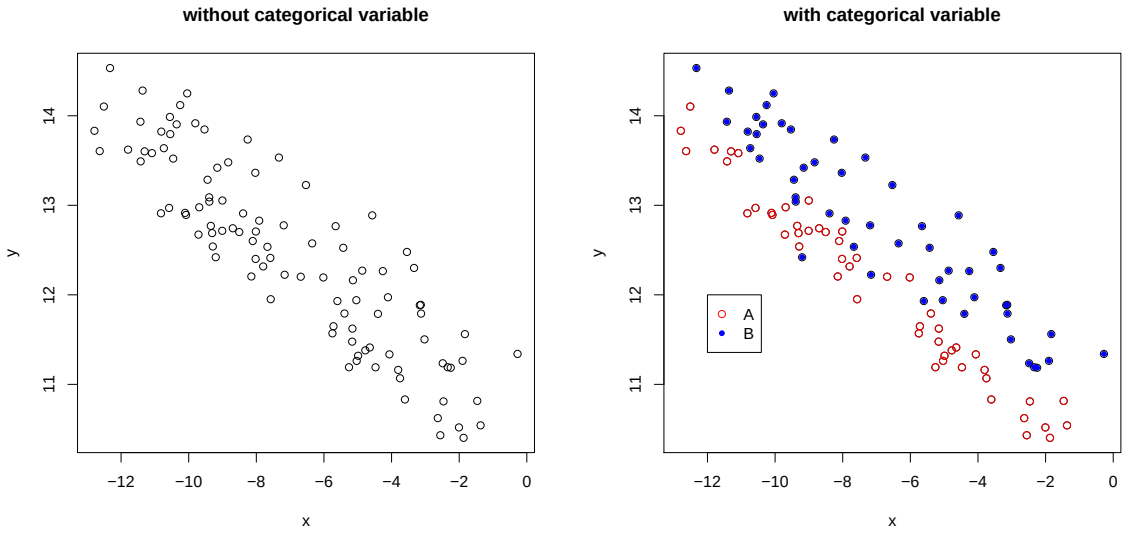
Outliers

On a scatterplot outliers are points that seem to deviation from the rest of the points, either by being far away from the general mass of the point cloud or clearly off the underlying form.



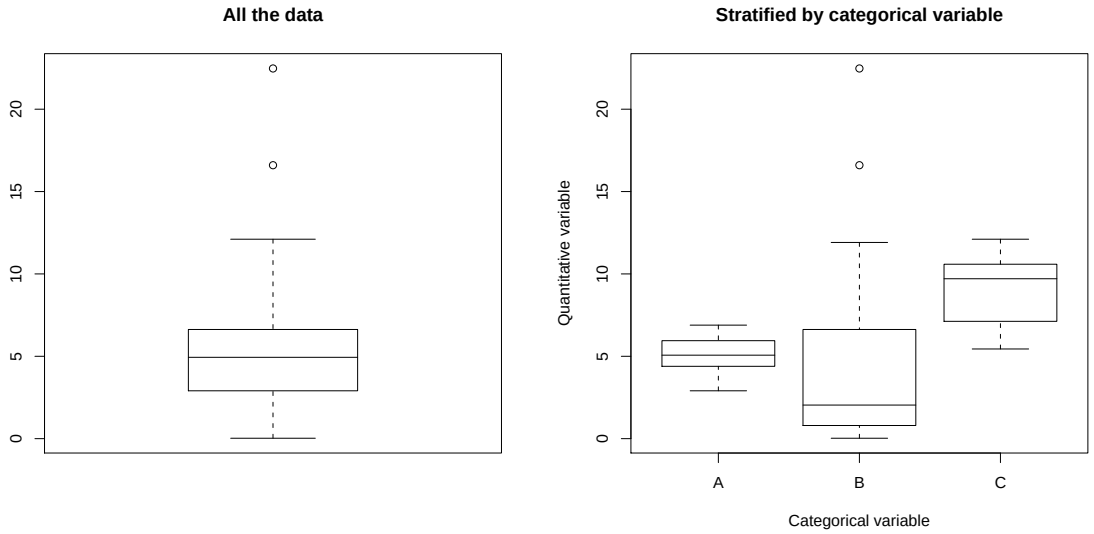
Adding a categorical variable to a scatterplot

Sometimes along with the two quantitative variables, we are also interested in the values of a categorical variable and its relationship to the two quantitative variables. We add the categorical variable to the scatterplot by using different plotting symbols or colors. Adding this to the graph may well add explanatory information to your plot.



Comparing a quantitative and a categorical variable using boxplots

Boxplots allow us to compare quantitative and categorical variables. Specifically, we draw boxplots of the quantitative variable where each boxplot is created using all individuals with a specific value for the categorical variable. Differences in the shape, size and position of the boxplots show whether there are differences in the quantitative variable between values of the categorical variable. In otherwords, it allows you see whether there is an association between the categorical variable and the quantitative variable.



Correlation

Correlation is a summary statistic which measures the strength and direction of the linear relationship between two quantitative variables each measured on the same individuals (or objects). Suppose our two variables are x and y. Then the formula for correlation is

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

where s_x and s_y are the sample standard deviations of x and y respectively. $\bar{x}$ and $\bar{y}$ are the sample means.

Properties of correlation

1. If $r > 0$ we say that there is a positive relationship between x and y
2. If $r < 0$ we say that there is a negative relationship between x and y

3. The smallest r can be is -1 and the largest is 1. ie $-1 \leq r \leq 1$

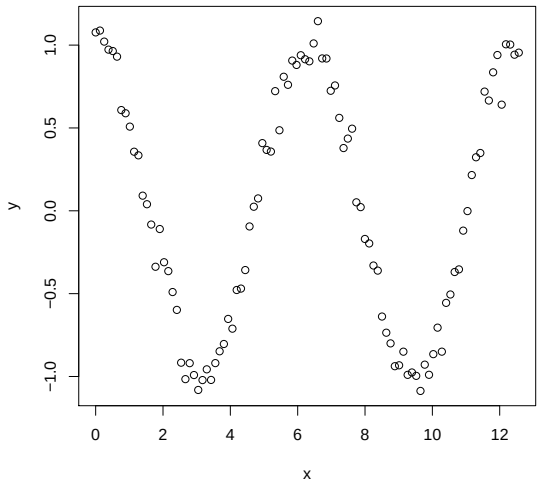
4. If the correlation is close to 1 we say that there is a strong positive linear relationship between the two variables

5. If the correlation is close to -1 we say that there is a strong negative linear relationship between the two variables

6. If the correlation is close to 0 we say that there is a weak relationship between the variables

7. Correlation has no units of measurement

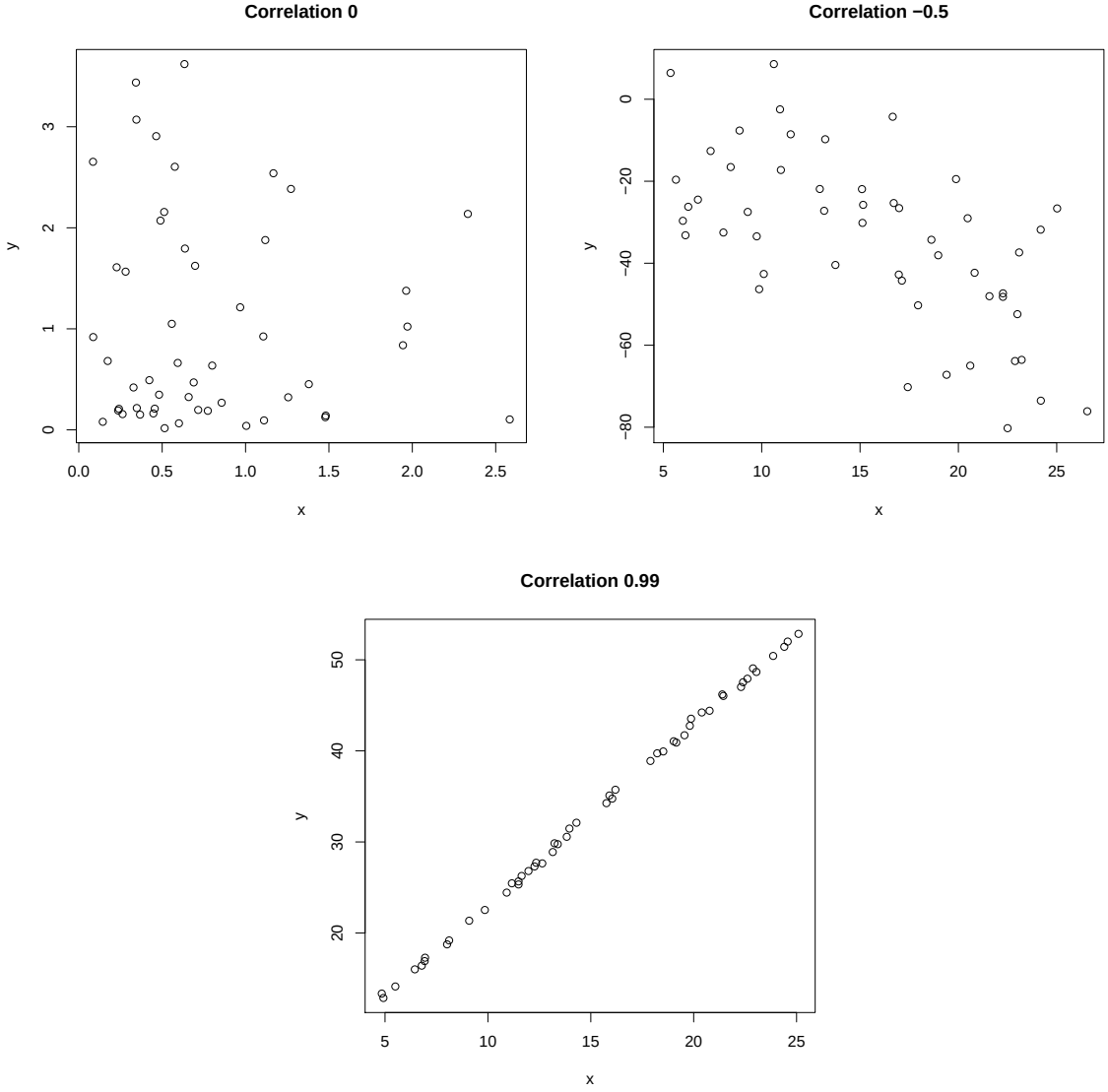
8. Correlation describes only the strength of the linear relationship. Non-linear relationships are not described no matter how strong. eg this situation would have correlation 0, but there is a clear and strong non linear relationship



9. Correlation can be affected by outliers

10. Correlation does not take into consideration which variable is the explanatory variable and which the response variable.

Some examples



An alternative formula for hand calculation of correlation

When computing correlation by hand you may find it easier to use this version of the formula

$$r = \frac{1}{n-1} \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{s_x s_y}$$