Homework #3 Solutions

4.13, 4.14, 4.20, 4.21, 4.30, 4.86

Problem 4.13

(a) The probability rules state that the total probability = 1. Therefore

$$P(O) + P(A) + P(B) + P(AB) = 1$$

$$\Rightarrow P(AB) = 1 - P(O) - P(A) - P(B)$$
$$= 1 - .45 - .4 - .11$$
$$= 1 - .96$$
$$= .04$$

(b) $P(\text{Can donate to Maria}) = P(O \cup B)$
$$= P(O) + P(B)$$
$$= .45 + .11$$
$$= .56$$

Problem 4.14

P(both have O blood) = P(American is O and Chinese is O)

= P(American is O) P(Chinese is O)

This is because of independence

= (.45)(.35)

= .1575

suppose that the first letter is American, second letter Chinese

$$P(\text{both same blood type}) = P(O_A O_C) + P(A_A A_C) + P(B_A B_C) + P(AB_A AB_C)$$

this subscript stands for the country

A = USA

C = China

$$= P(O_A)P(O_C) + P(A_A)P(A_C) + P(B_A)P(B_C) + P(AB_A)P(AB_C)$$

$$= (.45)(.35) + (.4)(.27) + (.11)(.26) + (.04)(.12)$$

$$= 0.2989$$

Problem 4.20

(a) To be a legitimate assignment of probabilities the total probability must equal 1.

$$0.000 + 0.003 + 0.060 + 0.062 + 0.036 + 0.121 + 0.691 + 0.027$$

$$= 1$$

so yes this is a legitimate assignment of probabilities

(b) $P(A)$ means "the probability that the randomly chosen American is Hispanic"

To be Hispanic you could be Black-Hispanic, white-Hispanic or Other-Hispanic. So

$$P(A) = P(\text{Black-Hispanic}) + P(\text{white-Hispanic}) + P(\text{Other-Hispanic})$$

$$= .003 + .06 + .062$$

$$= 0.125$$

(c) In words B is "the event that the person is white"

so B^C is "the event that the person is not white"

ie B^C is "the person is Asian, Black or Other"

$$P(B) = P(\text{Hispanic-white}) + P(\text{Not Hispanic-white})$$

$$= .060 + .691$$

$$= .751$$

$P(B^C) = 1 - P(B)$ (this is the complement rule)

$$= 1 - .751 = .249$$

(d) A = "Hispanic" $\Rightarrow A^C$ = "not Hispanic"

B = "white" $\Rightarrow B^C$ = "not white"

so "non hispanic white" is A^C and B

ie $A^C \cap B$

$P(A^C \cap B) = .691$ (from the table)

Problem 4.21

Events A and B are independent if and only if

$P(A)P(B) = P(A \cap B)$ so if we show that this is true then they are independent

From 4.20 $P(A) = .125$

$P(B) = .751$

From the table $P(A \cap B) = .060$

$P(A)P(B) = (.125)(.751) = .0939$

$\neq .06 = P(A \cap B)$

so A and B are not independent

Problem 4.30

P(Person is universal donor) = .07

P(Person is not universal donor) = .93

P(∅ of the 10 people are not universal donor) = $(.93)^{10}$

P(at least 1 of the 10 people is a universal donor)

$$= 1 - (.93)^{10}$$

$$= .5160$$

Problem 4.86

The addition rule says that

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

so

$$P(A \text{ or } B) = .134 + .254 - .080$$

$$= .308$$